# THE BOOK WAS DRENCHED

# LABORATORY MANUAL
# ENGINEERING PHYSICS
## Mechanics and Sound

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# PREFACE

The laboratory work in Engineering Physics at Stanford is but one phase of a carefully co-ordinated and synchronized course of study in physics. Although there may be one best way for an individual student to learn physics, there can be no one best way for a large heterogeneous class. It is evident, however, that there is a greater chance of a student's understanding a topic if it is put before him in more than one way. Accordingly, the course is arranged so that a student meets a given topic in a variety of ways: in reading assignments in the text, in demonstration lectures, in supplementary notes issued to students, in recitation and problem drill in problem sessions, in both the study and the performance of laboratory work, in homework problem sets, and in quizzes and examinations. These various types of presentation are synchronized so that, it is hoped, their impact on the student will have a maximum effectiveness.

Although in many universities the laboratory work is done in blocks of three experiments (so that a student's first experiment may be the one for the third week, and his third experiment may be the one for the first week), at Stanford the subject for a given experiment ordinarily is introduced in lecture the morning of the first day of the week during which all members of the class do that experiment. This correlation has resulted in students being able to do in the same length of time experiments considerably more substantial than those usually done. It appears that, as a consequence, students learn more for the same time and effort.

These experiments have grown up at Stanford over a period of decades. Dozens of people, too numerous to mention, have had a part in their development. In the postwar reorganization of the laboratory four essentially new experiments (7, 8, 9, 10) have been introduced and minor improvements have been made in all the others. The Laboratory Manual, of course, has been entirely rewritten. Experience with large classes, having many sections and many teaching assistants, has shown it necessary for effective instruction to have fairly complete laboratory instructions. Individual instructors, or students with permission of the laboratory instructor, may deviate from the material in the Laboratory Manual, if they wish to do so.

Suggestions for improvements in the experiments or in the Laboratory Manual will be welcomed by the author. It is probable that the instructors can understand the Manual, but the important thing is for the students to understand it. If parts are not clear, I shall be glad to have you point them out to me.

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INTRODUCTION

The laboratory work in Engineering Physics can be an exciting part of the course, or it can be drudgery, depending upon your attitude toward it. If you regard it merely as an impediment to your getting through the course, probably you will not enjoy it, and furthermore probably you will derive very little benefit from it. On the other hand, if you approach the laboratory with the thought that it is an opportunity to learn, and with a desire to make the most out of it, then it is almost certain you will find the time you spend on it both profitable and interesting.

It is true that you are not likely to be the discoverer of anything new in physics during your first-year course, for most (but not all) of the material in first-year physics has been known for decades; but it is also true that you have not known the material for decades, and you may, therefore, be able to experience the thrill in the laboratory of discovering for yourself some of the principles of physics. Most of the principles of physics were discovered by men using equipment no better than yours--in fact, most of it was not so good. We hope that you will be able to do some independent thinking about physical principles in the laboratory and that with the equipment in front of you, you will be able to try out your own ideas and find out things for yourself.

The experiments are intended not to be all alike. It is hardly possible to describe any one as typical because actually the experiments are quite different from each other and all have different purposes.

The experiment with the Force Table is primarily an exercise in trigonometry plus an attempt to show you how careful laboratory work (being careful about friction, etc.) can increase your accuracy. It is a simple experiment, but useful during the first week, before students have had an opportunity to study very much physics.

The experiment on Fundamental Measurements is used to teach you how to use certain instruments, and to impress on you the notion of precision (accuracy, significant figures, and errors).

The experiment on the Acceleration of Gravity is expected to teach you something about uniformly accelerated motion and also to give you some experience in deriving information from graphs.

The experiments on Centripetal Force, the Roof Truss, and Wave Motion and Sound are experiments illustrative of particular and important physical laws which sometimes seem vague to students until they have a chance to investigate them experimentally.

The experiment on the Rifle Bullet is intended to illustrate the principles of conservation of momentum and of energy (be sure you know which part of the experiment illustrates which) and, in addition, to suggest a practical application of physics.

The experiment on Rotational Energy and Inertia is a direct illustration of the fundamental principle of conservation of energy. It also provides a check on the torque equation.

The experiment on Periodic Motion offers not only a proof of Newton's second law (by the agreement of the experimental results and the theory based on the law) but illustrates graphically what happens in simple periodic motion--a very important kind of motion having numerous practical applications.

The experiment on the Torsion Pendulum permits you to discover some rather complicated physical principles for yourself by experiment. It is hoped that from this experiment you will get some feeling for the scientific method.

The experiments are not of uniform difficulty (they tend to increase in difficulty as the course progresses) nor do all of them take the same length of time to do. Experiment 51-2, although easy, takes a long time to do. Experiment 51-3 takes about three-quarters of one second to do plus a few minutes for preparation. The reports take varying lengths of time. Altogether a student should average taking not more than 5 hours per week to do the lab work and the report. If the student finishes the lab work early, he may expect to have extra time for the calculations and report. There is absolutely no excuse for a student ever having to spend more than 6 hours on a week's lab work.

In the preceding paragraphs we have discussed some of the more practical and tangible values to be obtained from the laboratory work. There are certain other values--less obvious but more important--that are also more difficult to acquire. It may be worth while to consider a few of them:

## An Understanding of the Attitude and Method of Science

The essence of the scientific method is this: Through observation the facts concerning a phenomenon are ascertained; from these a working hypothesis is formed, and from this hypothesis deductions are made concerning probable consequences resulting from varying the conditions associated with the problem. Experiments are then carried out to check whether the expected results are actually obtained and to see where the hypothesis needs revision or extension. The results of experiments are then classified and systematized in a way that leads to an understanding of the phenomenon considered and of related subjects. For an appreciation of the operation of the scientific method, one must study examples of its application. Although none of the experiments in the manual is an ideal example of the scientific method, since not enough cross-checking is done, each experiment may be considered as a partial example. Everyone knows the scientific method has been very fruitful in science. Can it be applied to social problems too?

## The Idea That a "Better Method" May Exist

Many measurements may be made in the laboratory in several different ways. Which way do you use, and why don't you use a different method? (Why do you hold a micrometer in the way discussed in the manual, or operate the centripetal force apparatus in a certain way, or tie the spring balances to the roof truss in a particular way?) In daily life and in their regular business many people always do things in their accustomed manner, never considering that there may be a better way; however, what was good for grandfather in grandfather's time may not be good for you today when the situations are different and new techniques are available for dealing with new problems. It is hoped that you can carry over into your daily life some of the characteristics of an inquiring mind, the development of which we trust will be accelerated in the physics laboratory.

## An Understanding of the Exactness and the Limits of Knowledge About Any Particular Matter

Many people are content with sloppy and vague ideas that are neither accurate nor precise. Reasoning from such an erroneous basis, they reach incorrect conclusions which they often support with vigorous but irrelevant argument. In this course you can--if you try--form the habit of examining the conclusions reached in each experiment to see if they are really justified, or if they are entirely justified but in a limited sense. (For instance, in the experiment on the force table, all forces act in one plane. Is anything done about forces in three dimensions? In the experiment to verify the relationship for centripetal force, do you investigate a variation in mass or in radius? How do these questions affect the conclusions?) The training in clear thinking you can give yourself in this laboratory may encourage you to think clearly in other matters also.

## Respect for the Prestige and Reliability of Authority

Consider as an example the experiment on the acceleration of gravity, in which it is likely that your results will be valid to an approximation of about 1 percent. There are, however, people who have determined the acceleration of gravity to one-thousandth of one percent. It may be surprising that these people can make their measurements and do their calculations in about the same time it takes you to do yours. They are experts in their field and know considerably more about the subject than you will when you finish the experiment. Unquestionably, you will accept their results and accord them the respect they deserve.

In other types of relations this respect for authority is often lacking. In matters of public policy, for instance, it is unfortunate that almost every layman considers himself competent to tell the experts in the government what to do about everything. The tragedy is that although we have had great statesmen (both Demo-

crats and Republicans) the number has been small. Furthermore, we often have had officials of limited vision whose very isolation in the government made them less competent than some intelligent laymen to determine public policy. In science it is clear that experts deserve their recognition. It should be the same in public affairs. To this end every citizen must share the responsibility of electing more experts (and fewer politicians) to government office, and then according them proper respect and support.

## Development of Character and Sense of Social Responsibility

There are many universities offering excellent courses of professional study. More important to you and to society than your professional training, however, are the standards of character you set for yourself. Unfortunately, no course of study is offered for development of character. The following ideas are presented with the thought that a short discussion of the matter is better than none.

Your lab report, for instance, should be an honest straightforward statement of your own ideas, not an attempt to sound as if you knew more than you do. Time was that a man had to build a better mouse trap to get people to beat a path to his door. Nowadays, some manufacturers with undeveloped senses of social responsibility, interested only in personal profit, try to get the cash register to ring more frequently by putting money into advertising propaganda directed at the credulous mass of people who believe anything they hear--even the worst radio commercials--than by building a better product. Standards of living for the consumers would be higher if development engineers and sales engineers would insist on dealing with facts rather than with fancy.

As another example, you are advised to do a clean job in the laboratory, be careful of your equipment, report possible damage, and leave equipment in an orderly condition, even though you may get away with it if you don't. The common welfare in the laboratory is served best when a give-and-take attitude exists rather than when each individual tries to get or to "hog" the best equipment for himself. It should be obvious how unfortunate it is in the world that more of this give-and-take attitude does not prevail. Many (although not all) individuals, corporations, and nations appear to be out for all they can get irrespective of the common good.

In a time when one man can fly around the world in three days, when one man can kill 100,000 people in a few microseconds with an atomic bomb, in a few days with a capsule of disease germs, or in a few months by poisoning or infecting their crops, the young engineer or scientist will do well to consider a resolution to be certain that, irrespective of personal or private profit, he will see to it that all his activities contribute to the positive side of the ledger of public welfare.

LABORATORY INSTRUCTIONS

## I. Purpose of the Laboratory

The student should give considerable thought to the question, "In what ways should I expect to benefit from the laboratory part of this course?" A partial answer is given here. The laboratory work helps you to understand and remember the physics you have studied; it gives you practice in the application of physical laws and logic to real cases, and in that way aids you to think clearly; and it gives you some skill in the use of scientific instruments and techniques. The laboratory also offers you the chance to rediscover for yourself some of the basic principles of physics, and, while performing actual experiments, it gives you a chance to try out some of your own ideas, to reason about the results, and to draw conclusions from them.

## II. Outline of Instructions

The laboratory work consists of five main parts: preparation, quiz, performance of the experiment, tabulation of data and approval by the instructor, and the report.

## III. Preparation

Before coming to the laboratory, the student must read the instructions for the scheduled experiment, including the references mentioned in the assignment sheet. This preparation is extremely important and is a prerequisite for successful performance of the experiment in the allotted time. If the preparation is done sufficiently in advance, the student may have the opportunity of raising questions in class regarding the experiment before coming to the laboratory.

## IV. What to Bring to the Laboratory

The student is expected to bring to the laboratory the Laboratory Manual, slide rule, pen, pencil, ordinary paper, graph paper, and, occasionally, colored pencils and books of tables.

## V. Lab Quiz

At the start of each laboratory period (or shortly thereafter) the student will be given a fifteen-minute, closed-book quiz based on the instructions for the scheduled experiment. The quiz may consist of several questions selected by the instructor from the following list or from other questions pertaining to the experiment (instructors please note underscored words).

1. In a single sentence state the main object of purpose to be achieved by performing the experiment.
2. In not more than three sentences outline the essentials of the method followed in the experiment.

3. In words or equations, or both, state the fundamental laws or principles involved in the experiment.
4. Define the following term, important in the experiment:_____.
5. List the observations which must be recorded in the experiment.
6. State which measurements in the experiment must be made with the greatest care and accuracy in order to avoid error in the final results.
7. Write and explain any equations used in the experiment. (Students should know all equations marked with asterisks.)
8. List and explain any precautions necessary in the experiment.

## VI. Performance

Possibly preceding but usually following the quiz there may be a short period of instruction, at which time the student should ask about any point that has not been made clear in his preparation. In many cases, however, no instruction is necessary and the student and his partner should check out necessary equipment and go directly to their apparatus and begin performing the experiment. The apparatus usually will be set up and ready for operation. As a general rule no more than two students should work at any piece of apparatus unless special permission is given by the instructor. It is hoped that the instructions are sufficiently clear that the student can follow them alone. The instructor is available in the laboratory, of course, for consultation throughout the period.

## VII. Care of Apparatus

Normal wear and tear of apparatus is expected, but loss or damage must be paid for by the person responsible. Much of the equipment is costly and delicate. Students are asked to treat it with care and to report at once anything wrong so that it may be corrected as soon as possible. Students must leave their equipment in a neat condition.

## VIII. Tabulation and Approval of Data

Data taken include the following:

1. Identification of all pieces of apparatus used. This may be done by noting the manufacturer, range, and stock number of each piece of apparatus. These data are of most use when part of an experiment must be done over.

2. All readings necessary for the completion of the experiment. These data should be tabulated neatly to be used later in the calculations. The Data Sheets of the various experiments are arranged in what is believed

to be a convenient form for recording data. Students are advised, however, that the Data Sheets should be considered only a guide and that it does not necessarily follow that spaces exist on them for all useful data, or that all spaces must be filled with entries. Students may use additional Data Sheets if they wish. The purpose of printing the Data Sheets is to enable instructors to save time in grading reports by having most of the students' Data Sheets fairly similar to one another.

3. Units for all numerical data. Lengths, for instance, are rather commonly measured in the following units: kilometers, meters, centimeters, millimeters, Angstrom units, mils, inches, hands, feet, yards, fathoms, rods, chains, furlongs, miles, and light years. A length of 6.84 without units is completely meaningless.

4. A statement of any unusual features which might affect the accuracy or interpretation of the results.

5. A brief description or diagram of a technique used for gathering data if this technique is not described in the Laboratory Manual.

6. Date of performance of the experiment.

Although ink is preferred, it is satisfactory to use pencil for data. Erasures never should be made on the Data Sheet; if a mistake is made, cancel the wrong part with a line. All in all, the data should be sufficiently complete that a person who has not done the experiment can write up a complete report from the data.

## IX. Report

The report should be written individually by the student in ink or preferably it should be typewritten. It consists of the Data Sheets (torn out of the Laboratory Manual), Calculations called for in the Laboratory Manual, together with sample calculations to show how the calculations were performed, Discussion called for in the Laboratory Manual, and the Graphs called for in the Laboratory Manual. Ordinarily, the instructor should not assign all the questions in the Discussion. It is expected that about one and one-half pages of discussion should be ample for most experiments. It is expected that reports will be written in good English, inasmuch as no student is admitted to regular work at Stanford University whose command of the English language is inadequate. (Students whose native tongue is not English will be accorded a certain degree of leniency.)

The student should strive to make his discussion concise, brief, and to the point. If he does not have a worth-while remark to make in answer to a question, he should omit that question. Brevity is a virtue.

The report should be handed in at the time (not less than four days after the performance of the experiment and place designated by the laboratory instructor, possibly in a suitable folder. Late work is accepted only by special arrangement with the laboratory instructor. In no case will the instructor keep the report for longer than seven days. Usually it will be graded and returned in three days or less. [It is requested that students report to the professor in charge of the course abuses (if any) by laboratory instructors.]

Students are advised to try to complete the laboratory experiment and the report in a total of not more than five hours. If they take longer than this they are not spending their time efficiently.

A good report is well written and concise; the answers to the questions are good answers. The calculations are made accurately and are well labeled, such as by underlining or boxing important results. Points may be lost in the laboratory for such things as:

(a) wrong arithmetic;
(b) using too many or too few significant figures;
(c) failure to indicate a greater accuracy in an average than in any one reading;
(d) wrong physics (e.g., subtracting vectors instead of adding);
(e) bad results (e.g., getting the density of aluminum to be $\frac{6700 \text{ Kg}}{m^3}$ instead of $\frac{2700 \text{ Kg}}{m^3}$);
(f) handwriting too fancy, illegible, or too faint;
(g) use of poor grammar;
(h) "sources of error" merely listed without discrimination as to which are important and which are unimportant;
(i) poor answers;
(j) leaving out answers;
(k) graphs or diagrams poorly drawn;
(l) axes of graphs poorly labeled or not labeled;
(m) lack of title and necessary information telling what a graph is about;
(n) omitting apparatus numbers or catalog numbers (especially of electrical meters);
(o) failure to put physics box number on report.

The Discussion part of the Report is important. It explains how well the student understood the experiment and how well he performed it; it provides the best means for differentiating between good and poor reports.

X. The professor in charge of the course will be ready at all times to discuss the course with the students. Suggestions and criticisms (favorable or otherwise) always will be welcome.

# MEASURING INSTRUMENTS

## INTRODUCTION

For some students these directions on the use of measuring instruments are superfluous, but most students will find them worth reading. Everyone is cautioned that many of the instruments are delicate and easily damaged. Use them carefully, because damage must be paid for by the person responsible for it.

## THE RULER AND PARALLAX

1. Always place the ruler on the object being measured so that the surface of the scale is at right angles to the surface of the object being measured. In this way the marks of the scale are brought into contact with the object being measured.

If the scale marks are some distance from the object, the reading on the scale depends on the position of the observer's eye. An apparent change in position of any object due to a change of position of the observer's eye is called parallax. In this special case the scale reading changes as the position of the observer's eye is changed. Parallax can be avoided only when the scale markings touch the object.

2. If the end of the ruler is worn, use the middle of the scale.

3. Always estimate readings to a tenth of the smallest division.

## THE VERNIER CALIPER

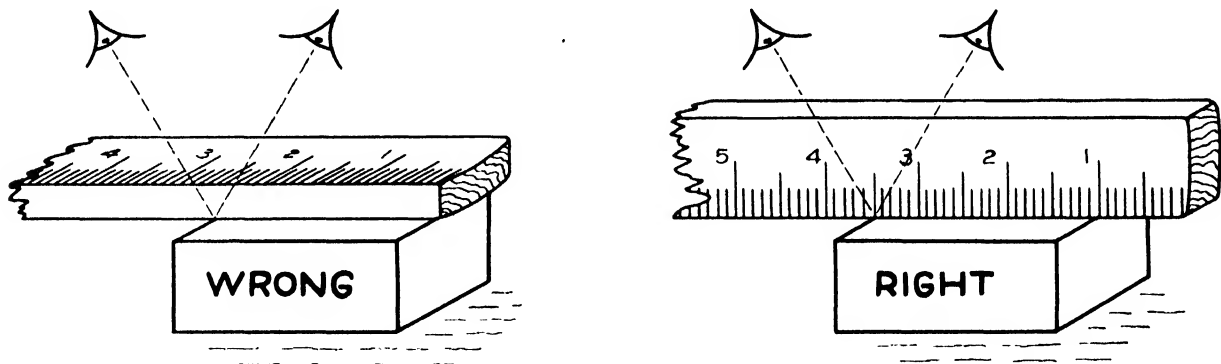1. The vernier caliper shown in Figure 2 may be



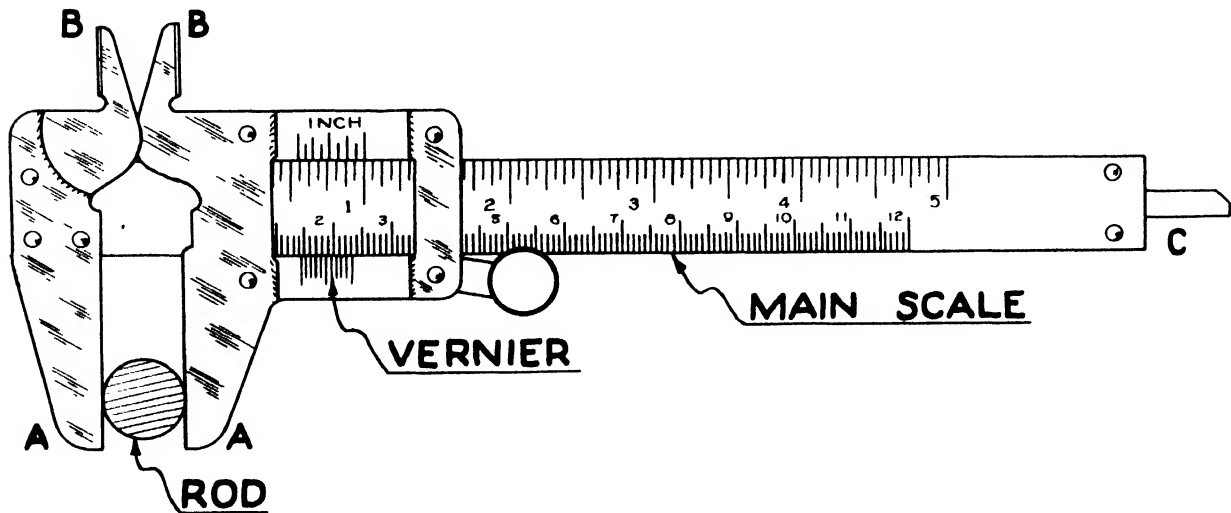Figure 1. The wrong and right ways to use a ruler.



Figure 2. The vernier caliper shows the diameter of the rod to be 14.3 mm.

used to measure the outside dimensions of an object using jaws AA, the inside dimensions of a hole using jaws BB, or a depth using depth gauge C. There are two scales on the caliper, the main scale which is fixed and the sliding or vernier scale (named after the Frenchman, Pierre Vernier, who invented it in 1631). By means of the vernier scale small fractions (usually tenths) of the smallest divisions of the main scale may be read directly without estimating.
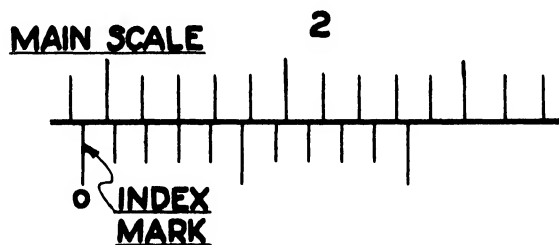
**MAIN SCALE 2**



o INDEX MARK

Figure 3. An enlarged view of the main scale and vernier scale

2. The index mark, or the zero mark on the left end of the vernier scale, is the index which is read against the main scale to determine the approximate dimension being measured. In the diagram, the diameter of the rod is somewhat more than 14 mm.

3. If the vernier is designed to read to tenths of the smallest divisions on the main scale, then each division on the vernier is only nine-tenths as long as a division on the main scale. In Figure 2, or the enlarged view in Figure 3, it is seen that the third vernier mark lines up exactly with a mark on the main scale. The second mark is therefore 0.1 mm to the right of a mark on the main scale, the first mark 0.2 mm to the right of a mark on the main scale, and the zero or index mark is 0.3 mm to the right of a mark on the main scale. Therefore, the diameter of the rod is 14.3 mm. Other dimensions are read in a similar way. Usually one needs a few minutes' practice in reading the vernier before he becomes proficient.

4. It may happen that when the jaws are closed, the caliper shows a zero-point error; that is, it does not read zero as it should. In this case it is necessary to make a suitable correction for other readings, corresponding to the zero-point error. For instance, if the caliper reads 0.2 mm with the jaws closed, then 0.2 mm should be subtracted from any other readings when they are made, but such caliper readings should be treated with some suspicion since the caliper obviously has not had good care.

5. It is good practice to measure the object as close to the main scale as possible, rather than out at the tips of the jaws.

6. When several successive measurements are made of the same dimension, the caliper always should be opened to some new value and then reset on the object being measured so that the first measurements

will not prejudice the results of successive measurements.

7. Some calipers are marked in the English units, some in metric units.

THE MICROMETER CALIPER

1. The various names for parts of the micrometer are shown in Figure 4. After the object to be measured is placed between the anvil and the spindle of the micrometer, the thimble is screwed up until the spindle and anvil are nearly touching the object. The final screwing up is done with the ratchet so that always the same final tightening force is used. The micrometer consists of a very carefully polished screw (not visible) fastened to the thimble and sleeve. The reading is actually determined by the position of the screw, and the accuracy of the reading depends on how carefully the screw is made. The micrometer may be read directly to millimeters by noting how much of the main scale on the hub is exposed. In Figure 4 this is seen to be something more than 2 mm.

2. The sleeve is uniformly graduated into fifty divisions. Since it takes two revolutions of the screw to advance the spindle one millimeter, every one of the fifty divisions on the sleeve corresponds to an advance of one one-hundredth of a millimeter. One determines by inspection whether the first or second fifty divisions are involved by noting whether or not more than half of the last millimeter is exposed on the hub. (Some micrometers, such as the one shown, have a marking every half-millimeter.)

3. In Figure 4 it is seen that more than half of the last (the third) millimeter is exposed, and that the reading on the sleeve is 36 divisions. Accordingly, the diameter of the wire is 2.86 mm. One may estimate to tenths of a division on the sleeve in order to read to thousandths of a millimeter.

4. Micrometer calipers usually cost from $ 3 to $ 15. Some are marked in millimeters, some in inches, and there are various modifications in different instruments. Micrometers that have been misused commonly have a large zero-point error which always must be checked and allowed for. If the object being measured is not at room temperature (for instance, it may be warm because it has just been turned on a lathe), the reading may be in error.

5. It is considered good practice to hold the object being measured in the left hand, to place the fourth (ring) finger of the right hand through the opening between the frame and spindle, and to work the thimble with the thumb and forefinger.

6. When several successive measurements are made of the same dimension, the micrometer always should be reset for each successive reading.

7. When not in use the micrometer spindle should be left at least a millimeter away from the anvil, as the screw of the micrometer is easily damaged if the micrometer is closed.
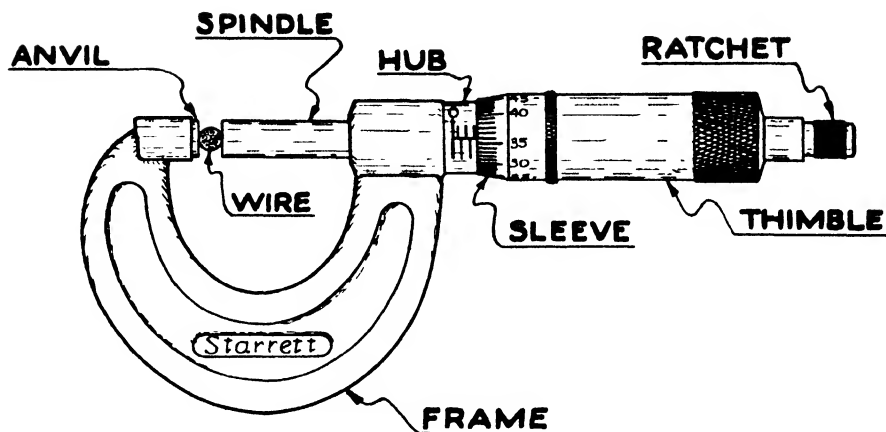
Figure 4. The Micrometer

## THE TRIP SCALE

The trip scale, sometimes called a platform balance, is shown in Figure 5. It is a device for weighing* objects to an accuracy of about 0.1 gram. Check to see that the scale balances when it is empty, and if it does not, adjust it so that it does. The unknown object is placed gently on the left pan, and the necessary standard weights are placed gently on the right pan. The exact balance is obtained by adjusting the position of the sliding weight or weights. The scale should be left unloaded when one is finished with it.
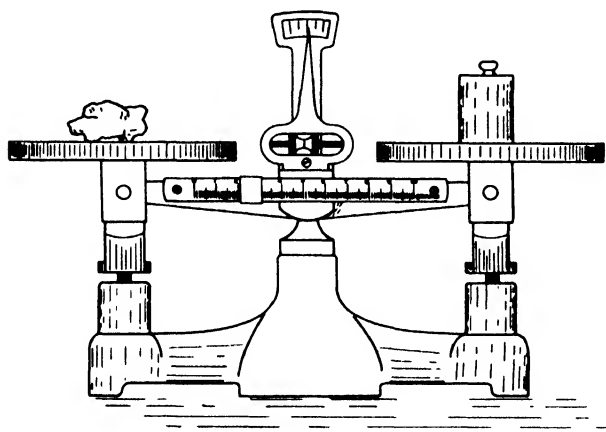
Figure 5. The Trip Scale

_____

* Strictly speaking, the trip scale compares the pull of gravity on the object and on the standards used as weights. Since the weight of the standards varies from place to place--being less by a fraction of a percent in the mountains than at sea level, for instance--it is perhaps more proper to say that the trip scale compares the mass of the object with the mass of the standard. Students are advised to ignore the subtle aspects of this paragraph until after they have studied Newton's second law of motion.

## THE ANALYTICAL BALANCE

1. The analytical balance shown in Figure 6 is more delicate than the trip scale, and it works differently. In this case the pans hang from the beam. The bearings of the balance have to be very sharp if the balance is to be accurate, and consequently they cannot support more than about 100 grams weight. Accuracy to 0.1 milligram is quite possible, corresponding to an accuracy of one part in a million. It is imperative that the balance be handled with care. A knob, usually on the front of the case, is used to lift
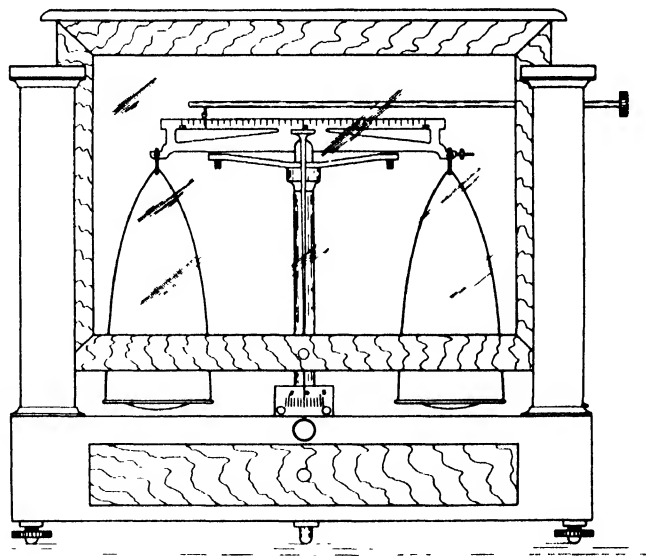
Figure 6. The Analytical Balance

the balance off the bearings whenever any weights are to be added or removed, or when an object is placed on the pan, or when one is finished using the balance.

2. The object is placed on the left pan, and the weights on the right pan. All analytical weights should be handled only with forceps, for the natural grease

from fingers changes their weight enough to put them badly off calibration.

3. It is quite tedious to wait until the balance comes to rest to see if it is exactly balanced, so that a more rapid and more accurate method, the method of swings, is in universal use. The marks on the scale at the end of the pointer are numbered, mentally if necessary, from 0 to 20, 10 being the balance point. With a given set of weights on the pan, the average of three successive swings to the right is taken, along with the average of the intervening two successive swings to the left. The average of the left and right readings is now taken; this average is the value at which the balance would eventually come to rest.

An arbitrary change of, say, 2 milligrams is now made in the balancing weight, and the process is repeated for the new balance position. The exact weight of the unknown object can be determined by interpolation as illustrated by the following example.

4. When the sum of the weights is 14.232 grams, the balance swings as follows:

| Readings on scale for swings to left | Readings on scale for swings to right |
|---|---|



5.1          17.6

5.2          17.4

5.2          17.2

         17.4

The ultimate balance point on the scale is evidently $\frac{(17.4 + 5.2)}{2}$ or 11.3.

Suppose a weight of 2.0 milligrams is added so that the sum of the weights is 14.234 grams, and the ultimate balance point is found to be 7.1. The addition of the 2.0 milligrams has changed the balance point by 11.3 - 7.1 or 4.2 divisions, so that the sensitivity of the balance is 2.1 divisions per milligram. (The sensitivity usually varies with the load on the balance.) The weight of 14.232 grams was too light by

11.3 - 10 = 1.3 divisions or $\frac{1.3}{2.1}$ = 0.6 milligrams.

The weight of the object is therefore 14.2326 grams.

5. Needless to remark, it is necessary to determine the balance point of the balance when it is not loaded, and either to allow for the fact that it may not balance at 10.0 on the scale, or else to adjust it so that it does balance there.

6. When accuracy to only a milligram is required, all the numerical work ordinarily can be done approximately and quite rapidly in one's head.

THE STOP WATCH

There are several different kinds of stop watches, but the general plan of operation is the same for all. To start the watch, one either pushes a knob or slides a lever; to stop the watch, one either pushes the same knob again, or slides the lever back; to reset the watch, one usually pushes a knob. If the watch is wound, the best way to find out how it works is to experiment gently. Some practice usually is required to synchronize the necessary motion of one's finger with the action to be timed. Naturally stop watches should be handled with care.

OTHER INSTRUMENTS

There are many other kinds of measuring instruments used in the laboratory, but many of them are combinations of those already discussed. For instance, a traveling microscope is a microscope fixed with cross-hairs that should be sharply focused on the object to be measured, the microscope being mounted in a carriage controlled by a micrometer screw. Since the carriage is often quite heavy, there may be some "back-lash" or lost motion in the screw, that is, the screw may give different readings depending on whether it is turned to the left or to the right. In this case it is necessary to approach the edges of the object being measured always from the same direction.

Whenever a student encounters an instrument new to him, he should find out how it is supposed to work, and then gently investigate how it does work.

## $\Sigma$, the Greek letter capital Sigma, meaning "the sum of"

If a number of similar things are to be added together, for instance, the sum of the forces exerted on a rope by the participants in a tug-of-war, we may be more interested in the sum of these things than in the individual things themselves. Thus, to save space in writing--and hence to save time in thinking--we may write $\Sigma F$ instead of $F_1 + F_2 + F_3 + F_4 + \ldots$, where $F_1$, $F_2$, $F_3$, etc., represent the forces exerted on the rope by persons 1, 2, 3, etc. The letter $\Sigma$ does not multiply F, as it might appear to an uninformed person, but merely stands for the words "the sum of." Thus, in an evenly balanced tug-of-war between "primed" and "unprimed" groups, $F_1 + F_2 + F_3 + \ldots$, $= F'_1 + F'_2 + F'_3 + \ldots$, and we may write $\Sigma F = \Sigma F'$.

## $\Delta$ and $\delta$, the Greek letters capital Delta and small Delta

These symbols are useful if a change in some quantity is under consideration. For instance, if the velocity of a car at some time is $v_1$ and at a later time it is $v_2$, then the change in velocity is obviously $v_2 - v_1$. It is sometimes convenient to write this expression as $\delta v$, where $\delta$ is not a multiplier, but stands merely for the words "the change in." Sometimes it will be found that the words "a small change in" or "a little bit of" will be more appropriate. Thus $\delta x$ may be "a small additional bit of x."

The use of these notations will become clear to the student after he has studied mathematics or physics for a while.

## Powers of 10

Because physics frequently deals with big numbers, it is convenient and particularly useful in extended computations to write these numbers using the appropriate power of ten multiplied by a simple number between 1 and 10. Thus:

$$
\begin{array}{rcl}
1{,}230{,}000 & = & 1.23 \times 10^6 \\
123{,}000 & = & 1.23 \times 10^5 \\
12{,}300 & = & 1.23 \times 10^4 \\
1{,}230 & = & 1.23 \times 10^3 \\
123 & = & 1.23 \times 10^2 \\
12.3 & = & 1.23 \times 10^1 \\
1.23 & = & 1.23 \times 10^0 \\
0.123 & = & 1.23 \times 10^{-1} \\
0.0123 & = & 1.23 \times 10^{-2} \\
0.00123 & = & 1.23 \times 10^{-3} \\
0.000123 & = & 1.23 \times 10^{-4}
\end{array}
$$

Any book on elementary algebra gives these rules for handling exponents:

$$ n^a \times n^b = n^{a+b} $$

$$ \frac{n^a}{n^b} = n^{a-b} $$

For instance:

$21{,}000 \times 300 = (2.1 \times 10^4) \times (3 \times 10^2) = 6.3 \times 10^6 = 6{,}300{,}000$

$210 \times 0.003 = (2.1 \times 10^2) \times (3 \times 10^{-3}) = 6.3 \times 10^{-1} = 0.63$

$210 \div 0.003 = (2.1 \times 10^2) \div (3 \times 10^{-3}) = 0.7 \times 10^5 = 70{,}000.$

## The solidus or diagonal line /, meaning "per" or "divided by"

We may write "miles per hour" as "miles/hour," or "mi/hr" or $\frac{\text{miles}}{\text{hour}}$.

# TRIGONOMETRY USEFUL IN PHYSICS

Trigonometry is the branch of mathematics that deals with the relationships between and among angles and sides of triangles, and in particular with the methods of solving for various of the quantities in terms of the others.

A right triangle is a triangle containing a right angle. It is a theorem in geometry that if two right triangles have an acute angle of one equal to an angle of the other, the triangles are similar and their sides are proportional, thus if angle $A$ = angle $A'$ the two right triangles in Figure 1 are similar and

$\frac{a}{c} = \frac{a'}{c'}$ also $\frac{b}{c} = \frac{b'}{c'}$ and $\frac{a}{b} = \frac{a'}{b'}$. In trigonometry these ratios are given forbidding-sounding names.

Thus, $\frac{a}{c}$ is called the sine of angle $A$, i.e., the number expressing the length of the side opposite the angle $A$ divided by the number expressing the length of the hypotenuse is called sin $A$; $\frac{b}{c}$ is called the cosine of angle $A$, i.e., the . . . length of the side adjacent to angle $A$ divided by the . . . length of the hypotenuse is called cos $A$; $\frac{a}{b}$ is called the tangent of angle $A$, i.e., the . . . length of the side opposite angle $A$ divided by the . . . length of the side adjacent to angle $A$ is called tan $A$. Thus $\frac{a}{c}$ = sin $A$; $\frac{b}{c}$ = cos $A$; $\frac{a}{b}$ = tan $A$; and of course it follows that $a = c$ sin $A$; $b = c$ cos $A$; and $a = b$ tan $A$. It is surprising how _useful_ these simple relations are.

Sin $A$ is equal to $\frac{a}{c}$ and cos $B$ is also equal to $\frac{a}{c}$, hence sin $A$ = cos $(90^\circ - A)$. Other similar relationships can be worked out by the student. The functions for angles larger than $90^\circ$ are discussed in trigonometry courses. A few useful relations are:
sin $(180^\circ - \varphi)$ = sin $\varphi$, and cos $(180^\circ - \varphi)$ = $-$cos $\varphi$.

The actual values of the ratios $\frac{a}{c}$, $\frac{b}{c}$, $\frac{a}{b}$ are given in tables. Thus, if $A$ = $30^\circ$, it is found that side $a$ is one-half as long as side $c$ (no matter how big the right triangle is), and in the table one finds sin $30^\circ$ = 0.500. It is worth remembering sin $30^\circ$ = cos $60^\circ$ = 0.500; cos $30^\circ$ = sin $60^\circ$ = 0.866; sin $45^\circ$ = cos $45^\circ$ = 0.707.

It is shown in books on trigonometry that in _any_ triangle ABC: $c^2 = a^2 + b^2 - 2ab$ cos $C$ (law of cosines) and $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (law of sines).
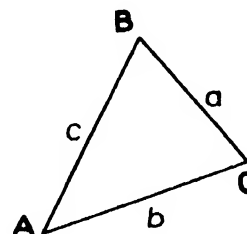


Figure 2. Any triangle ABC

Even this small amount of trigonometry will be a great help in the study of a first-year course in physics.

A typical problem (neglecting units) is this:
Given W (say W = 100), and angle $\vartheta$ (say $\vartheta$ = $30^\circ$), find P and N, these two forming sides of a rectangle, as indicated in Figure 3.
Clearly, P = W sin $\vartheta$ = 100 x 0.500 = 50

N = W cos $\vartheta$ = 100 x 0.866 = 86.6



Figure 1. Two similar right triangles ABC and A'B'C'



Figure 3. Typical Problem

## GRAPHS

1. There are several reasons for using graphs. One reason is that a graph is a pictorial diagram of the data that are presented. Oftentimes the information is grasped much more readily when it in the form of a graph rather than in the form of columns of figures. Another reason for using graphs is that by plotting a graph, inaccuracies in the data can be smoothed out by drawing a smooth curve near most of the points rather than a broken curve through all the points. Calculations based on the curve or line representing the best fit for all the points are more reliable than calculations based on only a few points. Still a third reason for using graphs is that by plotting appropriate quantities along the axes sometimes it is possible to develop or discover information not previously known. Some of the ideas expressed in this paragraph are discussed below in more detail.

2. Any graph shows the way in which two quantities are related to each other. One quantity usually is thought of as being independently variable (i.e., dependent on nothing else or chosen at will) and is called the independent variable. The other quantity that is thought of primarily in terms of its relation to the first variable is called the dependent variable. The values of the independent variable usually are laid off along the horizontal axis, in which case they are called the abscissae. The values of the dependent variables are then laid off along the vertical axis, in which case they are called the ordinates. When "y" is plotted as a function of "x" (plotting y against x), the values of y are the ordinates, and the values of x are the abscissae.

3. A smooth curve should be drawn near most of the points, distributing them fairly evenly on both sides of the curve. The curve usually does not go through all the points. If the curve is actually a straight line, it should be drawn as one straight line, and not as a series of short straight lines connecting the points. As mentioned above, one of the main reasons for using graphs is that inaccuracies in the data can be corrected for by plotting a smooth curve. In experiment 51-3, for instance, from measurements on a falling body, a graph of its velocity as a function of time is constructed. It happens that the graph is a straight line. From the graph 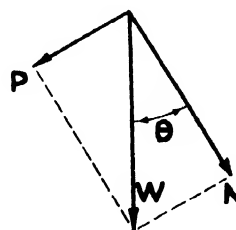the value of the acceleration of gravity g is determined. To calculate g it is shown that one must make use of two points on the graph. If one were to use two experimental points, any error or irregularity in either one would result in an erroneous value of g calculated from them. By using two points on the line, where the line represents the data with irregularities smoothed out, a value of g is obtained that depends on all the points taken together, and its value is greatly improved in accuracy.

4. As an aid in drawing smooth curves, it is often wise to put one's eye nearly in the plane of the paper and to view the curves at a glancing angle. (Put the graph in the center of a table, and then view it with your eye an inch or two above the edge of the table.) Turn the graph as may be necessary to see the curve "edge on." By this method it is possible to detect irregularities in the curve that would escape notice if it were viewed in the normal way.

5. The slope of a curve at a point is defined as being the slope of the straight line tangent to the curve at that point. The slope of a straight line is the ratio of the "rise" vertically, i.e., the difference in ordinates of two points on the line to the "run" horizontally, i.e., the difference in abscissae of the same two points. Thus if there are two points $P_1$ and $P_2$ on a line having the coordinates $x_1$, $y_1$, and $x_2$, $y_2$, then the slope of the line is:

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} \tag{1}$$

The slope is not the tangent of the angle of inclination unless the scales of ordinates and abscissae are the same. As an example of the slope of a curve, consider the equation of the velocity v, of a falling body as a function of the time t, if the body has an initial velocity $v_0$ . It is shown in lecture that $v = v_0 + gt$ where g is the acceleration of gravity. If v is plotted
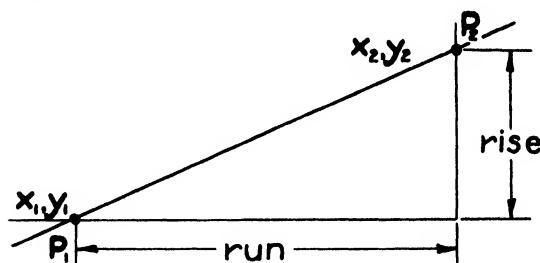


Figure 1. The Slope of a Line

as a function of t, one gets a straight line. To find the slope of the line, we select two times $t_1$ and $t_2$ when the velocity of the falling body is $v_1$ and $v_2$. We have, therefore, $v_2 = v_0 + gt_2$ and $v_1 = v_0 + gt_1$. On subtraction we get: $v_2 - v_1 = v_0 - v_0 + gt_2 - gt_1$. This yields

$$\frac{v_2 - v_1}{t_2 - t_1} = g. \tag{2}$$

By comparing Equations (1) and (2) we see that the left member of Equation (2) is the slope of the line represented by the equation $v = v_0 + gt$, so that evidently the slope of the line, $v = v_0 + gt$, is g.

6. A linear relationship is said to exist between two variables if a straight line is obtained by plotting one variable against the other on ordinary graph paper. A linear equation is a first degree equation. Continuing with the example of the previous paragraph, the two variables v and t are related by the first degree equation $v = v_0 + gt$ where $v_0$ and g are con-
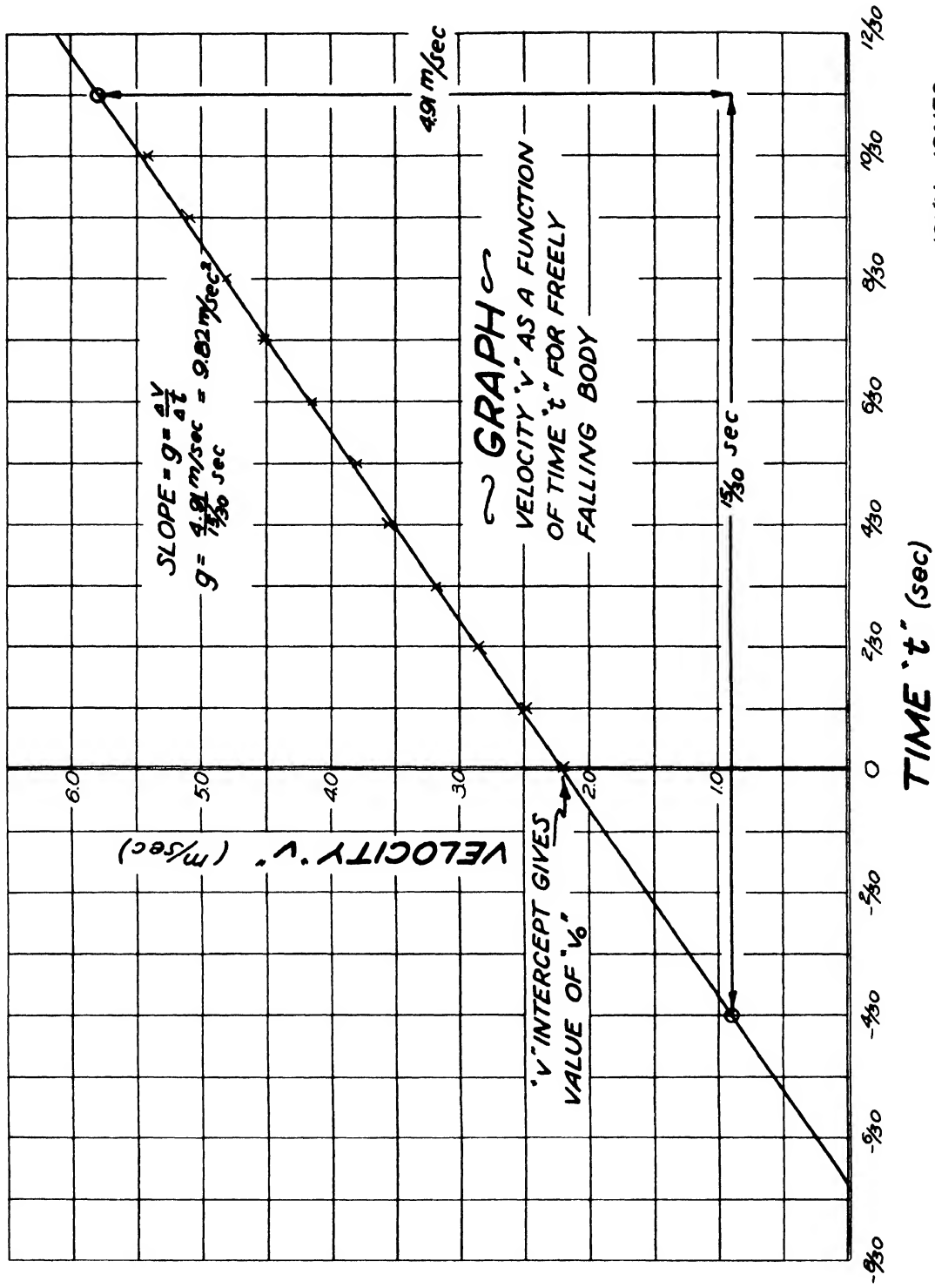
Figure 2. Example of a good graph.

stants. We have seen already that from the graph of v plotted against t, we may determine the value of g by measuring the slope of the straight line. It is easy to see that when $t = 0$, $v = v_0$ and hence the value of the other constant, $v_0$, may be determined from the v intercept, see Figure 2 on previous page. One may write the expression for the equation by making calculations on the graph. Thus $v = 0.220 + 9.82\, t$, from which we see that $v_0 = 0.220\, \dfrac{m}{sec}$ and $g = 9.82\, \dfrac{m}{sec^2}$.

7. As an example of developing information from a graph suppose that two quantities, x and y, are measured experimentally with the following results:

| x | 0 | 5 | 12 | 21 |
|---|---|---|----|----|
| y | 2 | 3 | 4  | 5  |

It is not known how x and y are related analytically. Suppose y is plotted as a function of x as shown in Figure 3. The points are seen to lie more or less on the straight line. One might conclude that x and y were linearly related, but if he examines the points more carefully he sees that they show a systematic deviation from being on a straight line. Suppose then that he plots $y^2$ against x.

| x | 0 | 5 | 12 | 21 |
|---|---|---|----|----|
| $y^2$ | 4 | 9 | 16 | 25 |

The result is shown in Figure 4. Since the points lie exactly on the straight line, it follows that a quadratic relationship exists between x and y. In fact by considering the x-intercept, it is seen that $y^2$ and y are zero when x is -4. From the slope of the $y^2$ versus x line, the slope is seen to be unity. The equation relating y and x is therefore $y^2 = 1 \cdot x + 4$ or $y^2 = x + 4$, and this equation obtained from the graph certainly represents more knowledge than the original series of numbers. Figure 5 shows a graph of $y^2 = x + 4$.

8. As a second example, consider the following data which have been obtained for quantities x and z.

| x | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| z | 0 | 1.72 | 10 | 27 | 55 | 96 |

It is seen at once that the graph goes through the origin. To find out if the data can be represented by an equation of the form $z = kx^n$, where k and n are constants take logarithms of both sides. $\log z = \log k + n \log x$. The last equation is the equation of a straight line of slope n when plotted on log-log paper. Although, when the data are plotted on ordinary graph paper, the curve has plenty of curvature; when the data are plotted on log-log paper, as shown in Figure 6, a straight line results. Its slope n is 2.5. When $x = 1$, $\log x = 0$, and hence the value of k may be determined as 1.72 from the z-intercept. Thus $z = 1.72\, x^{2.5}$, as can be verified from the data.

We see that on log-log paper a curve of the form $z = kx^n$ comes out as a straight line of slope n, and intersects the axis at $x = 1$ with the value k. (Note: If the data do not include the origin, the plotting process to get the exponent is more involved.) On semi-log paper a curve of the form $w = ab^{cx}$ where a, b, and c are all constants comes out as a straight line. When $x = 0$, then $w = a$, so the value of the w-intercept gives the value of the constant a. When one cycle (of 10) along the w-axis is equal in length to one division along the x-axis, then the slope is given by $c \log_{10} b$. Very commonly $b = e = 2.71828 +$ where e is the base of the natural system of logarithms, in which case the slope is 0.434 c.

A sine curve of the form $u = R \sin x$ where R is a constant, when plotted on polar coordinate paper, comes out as a circle of diameter R tangent to the axis at the pole.
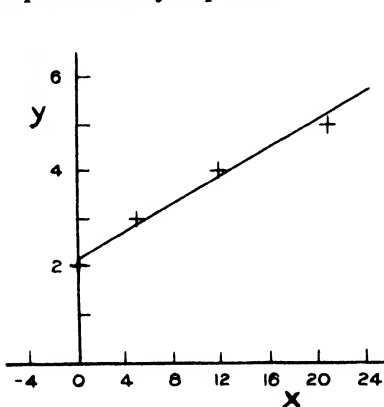


Figure 3. Graph of y plotted against x
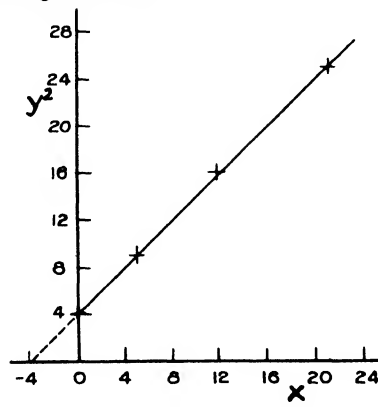


Figure 4. Graph of $y^2$ plotted against x
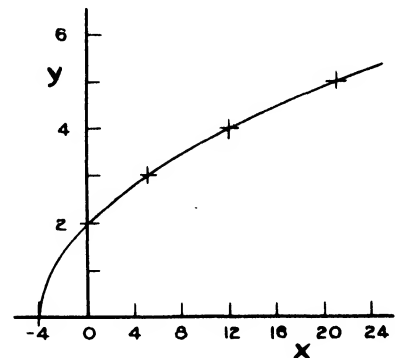


Figure 5. Graph of $y^2 = x + 4$

The student should familiarize himself with the possibilities of various types of graph paper.

## 9. RULES FOR DRAWING GRAPHS

a. Choose scales so that the experimental points extend most of the way across the graph paper both horizontally and vertically. The scales need not start at zero nor must they be linear. The bottom of the graph should be along the bottom or right-hand side of the paper--not along the left-hand side.

b. Place only a few well-chosen figures on the axes to indicate magnitudes.

c. Label the axes clearly and give units.

d. Indicate each pair of plotted values by a small x or + or circle. If two curves are plotted on the same paper, distinguish them by different marks. A dot is unsatisfactory as its position may be lost if a curve is drawn through it.

e. Draw a smooth curve near most of the points.

f. Give the graph a title.

g. State to which experiment the graph refers.

h. Give the date.

i. Put your name on the graph.

j. If there are points of particular interest, such as intercepts or points from which the slope of a straight line is figured, label these points. Show the calculations of slopes on the graph.



Figure 6. Plot of data on log-log paper. Slope is 2.5

# THEORY OF ERRORS

Nature of Measurement     Errors
Significant Figures     Propagation of Errors
Arithmetic by the "X" System

## I. Nature of Measurement

1. Making a measurement is a somewhat more complicated operation than is ordinarily realized. Suppose we consider a very simple case--measuring the height of a brass block. Suppose we have a good-quality steel rule. We place the block on a flat surface, place the end of the rule on the surface next to the block, and then compare the height of the block with the markings on the rule. The points to be emphasized are that we must compare the height of the block with the height of the marks on the rule and that this comparison involves an estimate on our part.

Other types of measurement also involve an estimate on the part of the investigator. Thus, to read a voltmeter, one compares the position of the pointer with the position of the marks on the scale. To weigh something, one must compare the position of the pointer on a balance with its scale, and so on.

2. Because all measurements ultimately involve an estimate by the person making them, no measurement can have infinite precision.

3. Returning to the example of the brass block, for a rough measurement occasionally it may seem that the edge of the block coincides exactly with a mark on the scale, but if one were to view the block and scale with a very high-power microscope, in all probability he would see that the coincidence was not perfect.

4. We may conclude, then, that different measurements are made with different degrees of precision, but that in no case is the precision perfect. In an economic cense it is very important to know how accurately measurements are made or what tolerances are permissible. For instance, suppose you want some steel blocks in the shape of two-inch cubes. If you require that the blocks be roughly cubic, having sides longer than 1.9 inch and shorter than 2.1 inches, the blocks can be sawed off rough from standard stock and you can buy the blocks for a few cents a pound. If you require that the blocks be cubic with sides longer than 1.999 inch and shorter than 2.001 inches, the blocks must be milled carefully by a machinist and will cost you several dollars a pound. If you require blocks longer than 1.99999 inch and shorter than 2.00001 inches, then the blocks must be lapped and polished, tested by a high-grade laboratory, polished some more, tested again, and so on. The blocks probably will cost you several thousand dollars a pound. In all cases the blocks are two-inch cubes, but the precision of the dimension is different in the several cases.
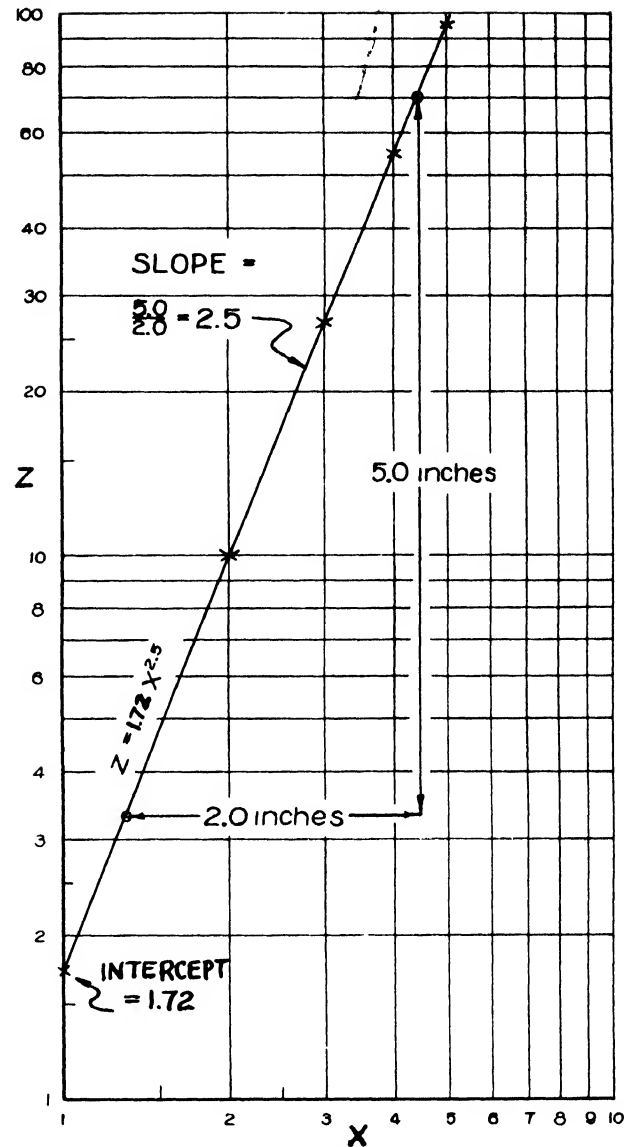
## II. Significant Figures

1. One of the simplest methods of expressing the degree of precision of a measurement is related to the idea of "significant figures."

2. All digits in a number are called significant figures except those zeros used to locate a decimal point.

Each of the following numbers contains 3 significant figures: 123, 1.23, 0.00123, 123,000, 1.23 x $10^7$, 101, 10.0, 0.0700. The number "100" contains one significant figure, since the zeros can be interpreted merely as decimal-point locators; but "100." contains three significant figures, as the position of the decimal point is shown and the zeros are not needed to locate it. The example of the steel blocks illustrates the fact that more significant figures are used to express a result with greater precision. In the first case the dimension is expressed to two significant figures; in the second case to four; and in the last case to six.

3. Now consider another example. If the width of a very rough undressed piece of lumber is measured and found to be about three and five-eighths inches--meaning somewhere between three and one-half and three and three-quarters inches--then it is wrong to write that the width is 3.625 inches, because such a statement implies that the width is known to be within about 3.624 and 3.626 inches, which is certainly not the case. If decimal notation is used, the width should be recorded as 3.6 inches, as the limit of the precision of the measurement is such that to add more figures implies greater accuracy than exists.

4. It is customary to record data under the assumption that the last significant figure in a number has been estimated, and may not be entirely reliable. For instance, in paragraph 3 the best value for the width might have been 3.5 or 3.7 inches. Thus, the 6 in 3.6 is not entirely reliable. Digits following the 6 are definitely unknown.

## III. Arithmetic by the "X" System

1. Pure mathematics by itself is one of those "useless" subjects like pure physics or music or literature or history, with which educated people should be familiar because the study of them helps to provide an understanding of the ways people in the world think and react and behave. When mathematics is applied to physical reality, however, it becomes a very valuable tool for the analysis of physical problems. The person who studies mathematics for its beauty and for its cultural value is to be encouraged if he enjoys the subject, but the engineer who uses mathematics for its practical value is advised to make sure he uses his tool in a practical way. A few examples are given below as to the proper methods of using arithmetic.

2. In the world today, speed has value, and obtaining excessive accuracy is wasteful of time and effort.

As an illustration of this matter consider the methods of obtaining the square root of numbers. In school one often is taught a method that looks like this (for the square root of 150).

```
            1 2, 2 4 7 4 4
        V‾150.0000000000
            1
  22        50
            44
  242       600
            484
  2444      11600
            9776
  24487     182400
            171409
  244944    1099100
            979776
  2449484   11932400
            9797936
            2134464
```

This method (which nearly everybody does not understand) is to be compared with the following method: guess the answer, divide it into the original number, and take the average of the quotient and the guess, thus: Everyone knows the square root of 144 is 12 and the square root of 169 is 13, so the square root of 150 can be guessed as 12.2.

```
                12,2950
        12.2 |‾150.0000
                122             12.2950
                280             12.2000
                244             12.2475 Average
                360                     or Answer
                244
                1160
                1098
                 620
                 610
                 100
```

The result is accurate to about twice the number of figures as the number to which the guess and the quotient agree. The method gives almost the correct answer and takes about half the time.

3. If you are esthetically inclined and care about the last decimal place, you must use the longer method, but if you are a practical person and merely want to know the answer as soon as possible, the second approximate method is preferred. Of course, even more rapid methods are to use a calculating machine if you have one, or to look up the answer in a handbook, or to use logarithms or a slide rule. The slide rule is not very accurate and will give you 12.24, but if that accuracy is sufficient, it is a waste of time and effort to use any method that takes longer.

4. In the modern world a man who can get the right answer quickly is a better man than the man who gets the same answer more slowly. It must be emphasized that the man whose answer is not sufficiently close to the right answer, however, is not worth anything, even if he gets his unsatisfactory answer in no time at all. Thus you should strive to get the answer of the required precision as quickly as possible.

5. Consider now a more common problem. Suppose you wish to know the distance between two points A and E as accurately as possible. The best measurement you find for the distance from A to B is 107.2 miles, from B to C is 2.063 miles, from C to D is 72 miles, and from D to E (which for some reason has been measured very carefully) is 263.192 miles. If one writes down the numbers and adds them he gets:

```
        107.2
          2.063
         72.
        263.192
        444.455
```

While it is true that the sum of the numbers is 444.455, is it proper to conclude that the sum of the distances is 444.455 miles? Definitely it is not proper, and the reason is simple. The 72-mile distance is known to a precision of not less than one mile, and definitely it is not 72.000 miles, or it would have been recorded that way. Since the decimal places for the 72-mile distance are unknown, nothing is gained by being concerned with them for other distances.

6. A useful system of arithmetic, seldom taught in schools, automatically indicates which digits may be ignored in arithmetical operations. It has no official name, but we shall call it the "X System." It is actually a very simple system. Unknown digits are recorded merely as "X," and one X is always put at the end of every number to indicate where the unknown digits begin. When the arithmetic is finished, all X's in the answer are replaced by (non-significant) zeros.

7. In the X System the addition for the previous problem looks like this:

```
        107.2XXX
          2.063X
         72.XXXX
        263.192X
        444.XXXX,   indicating a total of
                            444 miles.
```

8. Since there is no need to be concerned with unknown digits, we ignore all columns containing an X. We save time and trouble by NOT doing unnecessary arithmetic.

9. Consider the piece of lumber again. In the X system its width is 3.6X inches. Suppose its length is 101.X inches. What is its area?

```
        101.X
          3.6X
        XXXX
        606X
        303X
        36X.XXX     indicating an area of
                            360 square inches.
```

Note very carefully that although 3.6 x 101. = 363.6, the area of the board is 360 square inches and not 363.6 square inches, because the area of the board is known only to two significant figures.

10. The X System works as well for subtraction and division as for addition and multiplication.

11. If one tries multiplication, division, addition, and subtraction by the X System, he will find the following rules applicable:

In products and quotients the number of significant figures in the result should be equal to or be one more than the number of significant figures from which the product or quotient is derived.*

In additions and subtractions the number of <u>decimal</u> places in the result should equal the number of decimal places in the factor having the fewest decimal places.

IV. Errors

1. No physical quantity ever can be measured with infinite precision, i.e., to an infinite number of significant figures. Therefore, the true value of a physical quantity never can be known, although the uncertainty can be made very small by careful work. If a series of measurements is made on the same quantity, it is regarded as proper to consider the average as the best value.** In fact, the reason for taking the

---

* For example, a two-digit number when multiplied by a four-digit number should yield a product having only two or three digits as significant figures. Thus, in recording mathematical operations on numbers standing for physical quantities, 24 x 4005 = 96000, 24 x 5103 = 122,000. In the former case two significant figures appear in the result because the 9 in 96000 is a <u>larger</u> number than the 2 in 24, 24 being the less accurately known number, since it has fewer significant figures than 4005. In the latter case three significant figures appear in the result because the 1 of 122,000 is a smaller number than the 2 in 24. Since the use of 24 implies a range of uncertainty from 23-25, the uncertainty is about ±4 percent. The use of 96000 implies a range of 95000-97000, an uncertainty of about ±1 percent. The accuracy implied by the two significant figures in 96000 is then at least as good as the accuracy implied by the two significant figures in 24, so that two significant figures are sufficient. Had the result in the second case been given to only two significant figures such as is the case of 120,000, implying a range of 110,000 to 130,000, or a range of about ±8 percent, some of the accuracy implied by the use of 24 would have been lost. Consequently, three significant figures were retained in 122,000.

** This statement is strictly true if the observations follow a so-called Gaussian error function. The student had best ignore this matter until he is ready to study mathematical statistics.

average is that it is more precise than any one reading. Accordingly, a person is justified in keeping at least one and usually two more significant figures in the average of several readings (say three or more) than the number of significant figures that agree with each other among the various individual readings. If a metal block is measured six times and its thickness is found to be 5.13, 5.14, 5.12, 5.14, 5.15, and 5.12 cm, the average of which is 5.13333 cm, it is proper to regard the thickness of the block certainly as accurate as 5.13 cm. Note that all measurements agreed to 5.1 cm, and therefore the average is certainly good to 5.13 cm. One doubtful figure may be carried, giving the result as 5.133 cm (or in the X System, 5.133X cm).

2. The <u>uncertainty</u> in a measurement is called the <u>error</u> of the measurement. This use of the word "error" carries no implication of mistake or blunder; it means uncertainty.

3. There are three kinds of errors in physical measurements: systematic, accidental, and personal errors. Systematic errors arise from errors inherent in the equipment or the method. For instance, if a steel measuring tape is of the correct length at a certain temperature, it will be too long at a higher temperature. Measurements based on the assumption that the tape is of the correct length at the higher temperature will be in error.

Systematic errors often can be eliminated by careful work or by correcting for them. In the example given, one may measure the temperature, calculate the expansion of the tape, and make allowance for it.

Personal errors are personal to the investigator and need not concern us very much. It is known, for instance, that not all people will agree on certain kinds of color-matching measurements, or in synchronizing the push of their fingers on a stop watch with an event being timed. Ordinarily with experienced observers, personal errors are significant only in the cases where extreme precision is required.

Accidental errors are errors of chance. If several measurements of a quantity are made, ordinarily not all of them agree precisely. If no reason can be assigned to explain why any given reading is different from any other, other than inaccuracies beyond control of the observer, presumably the errors are accidental or random, and obey the laws of statistics. The remainder of the discussion of errors is concerned only with accidental errors.

4. The <u>probable error</u> of a physical quantity is that range in its value such that there is a 50-50 chance the average of an infinite number of observations of it would lie within the range. Thus, if the symbol ± refers to the range of the probable error (which is the common meaning of the symbol, although sometimes other meanings are used), and if the weight of an object is specified as 11.23 ± 0.07 lbs, there is a 50-50 chance that the true weight, or the average of an infinite number of measurements, lies within 11.16 and 11.30 lbs. Of course, there is

also the 50-50 chance that the weight is more than 11.30 or less than 11.16 lbs.

5. A residual is defined as being the difference between any observation and the average of a series of observations on the same quantity. A rather involved theory in statistics leads to the conclusion that the probable error of the mean or average of a series of measurements on a quantity, which we may call $PE_{av}$, is given by

$$PE_{av} = 0.6745\sqrt{\frac{\text{sum of the squares of the residuals}}{(\text{no. of observations})\ (\text{no. of obs.} - 1)}}$$

Strictly speaking the theory referred to in paragraph 5 is applicable only if several dozen observations have been made. In Engineering Physics one often does not make that many observations on any one quantity, and, as the probable error is tedious to calculate, we shall define a simpler quantity which is nearly equal to it and is much simpler to calculate. It is called the probable average deviation, or PAvD, and is equal to the average deviation of the observations from the mean divided by the square root of the number of observations. An example is given in the next paragraph.

6. The observations of the time for a certain event to occur are indicated below, together with the calculation of the probable error of the average and the probable average deviation.

| | Time | Deviation from Mean, or Residual | | Square of Residual |
|---|---|---|---|---|
| | 17.2  sec | 0.1  sec | | 0.01 |
| | 17.3 | 0.0 | | 0.00 |
| | 17.5 | 0.2 | | 0.04 |
| | 17.5 | 0.2 | | 0.04 |
| | 17.0 | 0.3 | | 0.09 |
| Average or Mean | 17.30 | Av. Dev. from Mean | 0.16 | Sum 0.18 |

$$PAvD = \frac{0.16}{\sqrt{5}} = 0.07$$

Result = 17.30 ± 0.07  sec

$$PE_{av} = 0.6745\sqrt{\frac{0.18}{(5)\ (4)}}$$

$$= 0.6745 \times 0.095$$

$$= 0.064$$

Result = 17.30 ± 0.06 sec

The results are seen to be nearly the same.

7. The symbol "±" is used usually to indicate probable error, sometimes to indicate probable average deviation, sometimes to indicate the limit of error--the last term implying "certainty" that the "true" value lies within the range specified--and some-

times to indicate merely an estimate of errors. This vagueness as to the meaning of ± is unfortunate.

8. In statistics a quantity known as the standard deviation is used often. It is the same as the expression for the probable error, except with the factor 0.6745 omitted. At Stanford University the Registrar's suggested criterion for determining which students make a grade of A is that the students should exceed the class average of the passing students by an amount more than the standard deviation. In a typical case this means that about 15.9 percent of the passing students in a class get A's. The same criterion is applied (in the opposite sense) to determine the number of students who receive D's.

9. It stands to reason that if several sets of measurements have been made on a quantity, the set showing the smallest probable error is likely to be the most reliable. One must, however, never overlook the possibility of the existence of large systematic errors.

## V. Propagation of Error

1. From the discussions in the section on the X System it is clear that an uncertainty in any quantity is propagated through any calculations depending on that quantity. Thus an uncertainty in the width of the board introduces some error or uncertainty into the area for the board. The uncertainty in the length of the board introduces an error into the area also. Perhaps the value chosen for the width might have been too great and the value chosen for the length might have been too small. In that case the result for the area would have been more nearly correct than if both width and length had been in error in the same sense.

2. The theory of propagation of errors is too involved to give here in detail, but it has been established that:

I. The percentage (or fractional) uncertainty or probable error in the product (or quotient) of several numbers is equal to the square root of the sums of the squares of the percentage (or fractional) uncertainties or probable errors in the factors.

II. The percentage (or fractional) uncertainty or probable error in the product (or quotient) of several numbers containing a factor raised to the n power should be figured by taking n times the percentage (or fractional) uncertainty in that factor before squaring, adding, and taking the square root as given by rule I.

III. The absolute value of the uncertainty or probable error in the sum (or difference) of several numbers is the square root of the sum of the squares of the absolute values of the uncertainties or probable errors in the factors.

3. An example will illustrate the rules in paragraph 2. Suppose the diameter of a cylinder is found to be 5.530 ± 0.055 cm, and the height, 11.20 ± 0.22 cm.

What is the volume of the cylinder? Evidently the uncertainty in the diameter is 1 percent, and in the height 2 percent. The volume of the cylinder is $\frac{\pi d^2 h}{4}$. There is no uncertainty in $\pi$ or in 4, so that percentage uncertainty in the volume is $\sqrt{(2 \times 1\%)^2 + (2\%)^2}$ = 2.8%. The volume is found to be 269 cm$^3$ so that the result may be given as 269 ± 8 cm$^3$. If to this volume is added another of 100 ± 6 cm$^3$, the uncertainty is $\sqrt{8^2 + 6^2}$ = 10 cm$^3$, so that the total volume represented is 369 ± 10 cm$^3$. Notice that the error in the diameter of the cylinder is more important than the error in the height, because the diameter is squared and hence is doubled before it is squared and put into the formula for the error.

4. In doing extended computations, students should be careful not to waste time by carrying figures that are not significant. Conversely, they should be careful not to throw away their accuracy by dropping too many figures. Either the X System should be employed in making calculations, or some intelligent judgment should be used.

5. It is expected that students will apply the material discussed in Theory of Errors whenever it is applicable.

EXPERIMENT 51-1
## COMPOSITION OF VECTORS

### OBJECT

With the aid of a force table to test the validity of certain analytical and graphical methods of combining vectors.

### APPARATUS

In Lab: Force table ($ 30).

Check out: 4 pulleys, center pin, ring, cord, weight-holders, slotted weights, ruler, protractor.

### THEORY AND METHOD

A. A scalar quantity is one that has magnitude only, that is, one that is completely specified by a single number and a unit. Thus volume is a scalar quantity, for instance, a volume of 6 cubic feet is a scalar; some other examples are power, quantity of heat, pressure, and electric potential. A vector quantity is one that has magnitude and direction, and it turns out that it requires three numbers and a unit to specify it completely. It may be specified in various ways, but one way is to give its magnitude, and to specify the direction by giving its azimuth and elevation. Thus force (meaning push or pull) is a vector quantity; for instance, a force of 60 pounds horizontally is a vector; some other examples are velocity (magnitude and direction), torque, and electric field strength.

B. You are already familiar with the process of adding scalars; thus, if you have a refrigerator having a volume of 6 ft$^3$ and you obtain a second one with a volume of 8 ft$^3$, the total volume is 14 ft$^3$. On the other hand, if an object is being pulled north with a force of 6 pounds and east with a force of 8 pounds, the result is the same as though a force of 10 pounds were being applied in a direction N 53$^o$ 8$'$ E, or some what east of northeast.

C. In this experiment a force table is used to study methods of adding vectors. The force table consists of a horizontal circular plate about 16 inches in diameter, graduated in degrees, and provided with a set of movable pulleys at its circumference. By this means known forces can be applied to a small ring above the center of the table in any known horizontal direction by tying cords to the ring, running the cords over the pulleys, and hanging known weights from the ends of the cords. If the pulleys are of good quality, the frictional forces they exert are small, and the weight hung from the end of a cord may be taken as being the force exerted on the ring at the point where the cord is tied to it.

D. The results of graphical and trigonometric methods for the composition of vectors will be compared with experimental determinations for the following two problems:

Problem I. Find the vector sum of two forces, A = 150 grams weight, and B = 200 grams weight, making an angle 75$^o$ counterclockwise from A.

Problem II. Find the vector sum of three forces, A = 150 grams weight, B = 200 grams weight, making an angle of 75$^o$ counterclockwise from A, and C = 100 grams weight, making an angle of 295$^o$ counterclockwise from A.

E. The graphical and trigonometric methods for combining vectors are discussed in the text.

In the case of the polygon method for adding vectors, each vector in turn after the first one is slid around so as to maintain its direction until its rear touches the head of the preceding one. The resultant is then the vector connecting the rear of the first vector to the head of the last vector. Thus in Figure 2, A + B + C = D.

It is sometimes convenient to resolve a given vector into components. A vector may be resolved into innumerable combinations of components so that the sum of the components is equal to the original vector, but ordinarily the most useful type of resolution occurs when vectors are resolved into two components at right angles to each other, such as along horizontal and vertical axes. Thus, in Figure 3 the components of each A, B, and C are taken, then added together to get the components of D, which then may be constructed. Frequently the method of components turns out to be the fastest method of calculation.
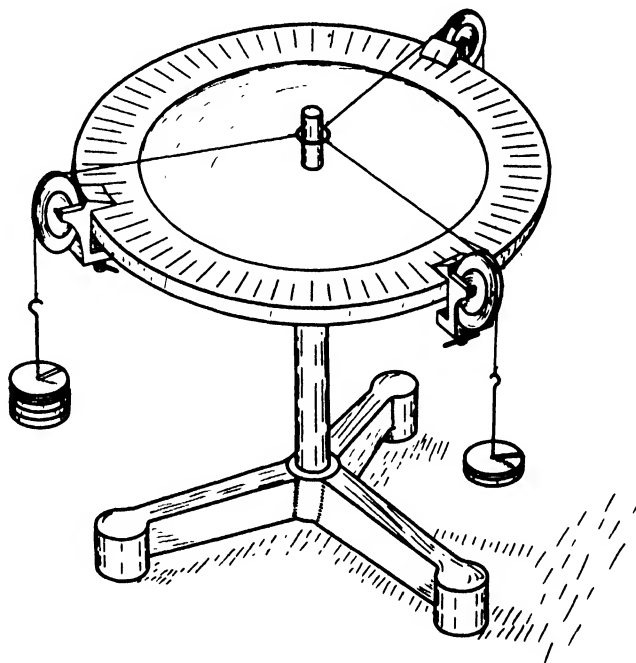


Figure 1. Force Table

In a trigonometric solution when the polygon method is used, ordinarily one must use the law of sines and the law of cosines. When the method of components is employed, use of the simple trigonometric relations involving sines and cosines is sufficient.

It should be pointed out that, although a vector is a directed quantity, it may be specified without saying anything about where it is. Thus, a velocity of 10 miles per hour east is the same vector whether it be in St. Louis or New Orleans.

3. As discussed in section D of the Theory and Method, tie three or four cords to the center ring and set up the various forces required, placing the pulley for force A in the direction marked $0^\circ$. Be sure to allow for the weight of the hangers.

4. By a process of intelligent successive approximations determine the direction of the equilibrant, and also its magnitude. As you approach equilibrium, it is important to keep the weights from swinging wildly. When equilibrium is achieved, the center of the ring will be directly over the center of the table.
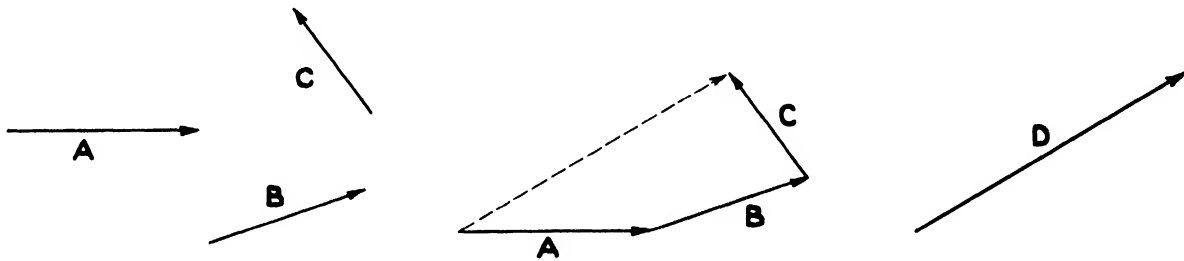
Figure 2. Vector addition by the polygon method of A, B, and C to form D, i.e., A + B + C = D.
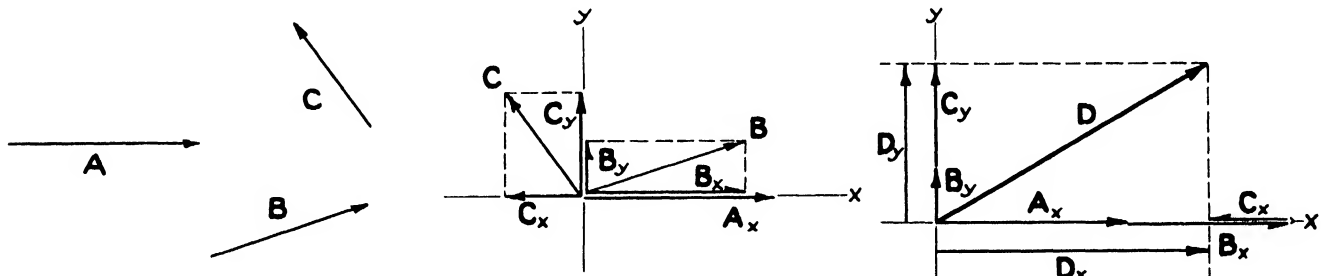
Figure 3. Vector addition by the method of components of A, B, and C to form D. Note that $D_x = A_x + B_x + C_x$, and $D_y = B_y + C_y$ (since $A_y = 0$), and that $D = D_x + D_y$.

F. The experimental determination is based upon the fact that if an object remains at rest, the vector sum of all the forces acting on it must be zero. If the sum is not zero, the object will start to move in the direction of the net force. The sum of the two forces in Problem I and the three forces in Problem II is called the resultant, or net force.

A force equal in magnitude to the resultant but opposite in direction to it, that balances it, is called the equilibrant, because the equilibrant of a group of forces holds the body on which they act in equilibrium.

PROCEDURE

1. Level the force table if necessary. Sufficient accuracy for the present purpose can be obtained by lining up the vertical support of the force table with two adjacent vertical corners of the room.

2. Place the center pin in the table and place the ring over it.

If you displace the ring slightly in any direction by pushing it with your finger, it will tend to return to the center. It will not show an equal tendency to return to the center from all directions, but only from any pair of opposite directions. In fact it is only by noting the equality of the tendency of the ring to return to the center when displaced slightly in pairs of opposite directions that a good balance can be obtained.

5. Be careful to see that the directions of the strings are such that if they were projected backward from the pulleys, all of them would intersect at the center of the ring. You may have to slide some of the strings a little way around the ring to accomplish this.

6. With care you can get a balance to about one or two grams weight and to about $0.5^\circ$ to $1^\circ$. When you are satisfied that you have the best balance you can get, start adding (or subtracting) weights in one-gram steps, until you are certain the balance is no longer

correct. For instance, if you should find 287 grams weight for the equilibrant, and you are sure that 284 is too small and 290 too large, then record your value for the equilibrant as 287 ± 3 grams weight. In this case the symbol ± specifies your limit of error.

7. Likewise determine your limit of error in direction, e.g., 27.2° ± 0.8°.

8. Make the required graphical and trigonometric solutions of the problems as indicated on the next pages.

9. Tabulate all your results on Data Sheet.

10. Leave your apparatus in a neat and orderly condition.

## DATA AND CALCULATIONS

a. Experimental data may be recorded directly on the Data Sheet.

b. Solve Problem I graphically (using either the parallelogram method or the polygon method) starting from point O, on page 5, and laying off vectors representing the magnitude and direction of the given forces A and B. Construct vector A on the given line with O at the tail of the arrow. Use a scale such that 1 cm corresponds to 20 grams weight. Make the constructions carefully, and measure the magnitude and angle of the equilibrant. Record results on page 4.

c. Solve Problem II in the same fashion, starting from point P.

d. On a separate page, added after the last page and attached to both pages of the Data Sheet, solve both problems trigonometrically by the polygon method, and record your results on page 4.

e. On another separate page solve both problems trigonometrically by the method of components and record your results on page 4.

f. Be sure to give units in your summary, and also specify whether the required directions are measured clockwise or counterclockwise from the direction of force A.

## DISCUSSION

On a separate sheet, clipped (not pinned), to the sheets of this experiment, write not to exceed one page in answer to as many of the following questions as the instructor assigns.

1. What basic ideas are illustrated by this experiment?

2. What conclusions do you draw from your experimental results? (If you make any comparisons, be quantitative.)

3. List a few of the important sources of error in this experiment, and make some quantitative estimate of the importance of each. (To answer this question well, you may wish to obtain and record somewhere on the Data Sheet some data not specifically asked for. For instance, you may wish to weigh the ring and strings; or you may set equal weights on the ends of a single string going over two pulleys 180° apart, and then determine what unbalanced weight is needed to overcome friction.)

4. If it were up to you, how would you improve the experiment, or do you think the experiment is worth while as is?

5. Why must the force table be level?

6. What is the reason for step 5 in the Procedure?

7. Comment on the question of whether a satisfactory balance could be obtained using only three forces of 100, 200, and 400 grams weight.

8. Velocity is a vector quantity. Therefore, velocities add like vectors. A man on a train going 48 miles per hour south, tosses a mail pouch to a man on a station platform by tossing the pouch 20 miles per hour at right angles to the direction of motion of the train (e.g., to the west). With what velocity--give magnitude and direction--does the pouch strike the man on the platform?

Name_____Box No._____Date in Lab_____

Partner_____Lab Section_____Instructor_____

## DATA SHEET

### Summary of Results, Problem I

| | Experimental | Graphical | Trigonometric (Polygon) | Trigonometric (Components) |
|---|---|---|---|---|
| Magnitude of equilibrant | _____ | _____ | _____ | _____ |
| Direction of equilibrant with respect to A | _____ | _____ | _____ | _____ |

### Summary of Results, Problem II

| | Experimental | Graphical | Trigonometric (Polygon) | Trigonometric (Components) |
|---|---|---|---|---|
| Magnitude of equilibrant | _____ | _____ | _____ | _____ |
| Direction of equilibrant with respect to A | _____ | _____ | _____ | _____ |

Other data:

Approval_____

Hours spent on this experiment: Study_____Lab_____Report_____

Graphical Solution to Problem I:

                                                                    forward direction of **A**
O  +- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - +- - - - - - - - - - - - - - - - - +- - - - —

Graphical Solution to Problem II:

                                                                    forward direction of **A**
P  + - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - + - - - - —

Note: Show your trigonometric solutions on pages inserted directly after this one.

EXPERIMENT 51-2
FUNDAMENTAL MEASUREMENTS

OBJECT

To teach the student to use the various instruments for measuring the fundamental quantities of length, mass, and time; to show that the average of several measurements is more reliable than any one measurement.

APPARATUS

In Lab: trip scale ($ 20), analytical balance ($ 100), clock with the second hand, bifilar pendulum consisting of a bob suspended from strings 3 meters long or longer, special board with scratches in nail heads about 3.95 m apart, special 4-meter stick.

Check out: ordinary centimeter scale, vernier caliper, micrometer caliper ($ 8), special aluminum block with hole near one side, steel ball, 3 brass blocks with thickness stamped on them, spring scale (250 grams), special awkward 10-cm ruler; stop watch ($ 25) check out separately.

THEORY AND METHOD

A. The fundamental unit of length is the meter. The centimeter is, of course, one percent of or one one-hundredth part of the length of the meter. The meter was originally intended to be one ten-millionth part of a quadrant of the earth's circumference. Actually the original determination of the size of the earth was somewhat in error so that the standard of length for the entire world is the distance between two fine scratches on a standard meter bar kept in a vault near Paris. This distance has been compared with great precision with the wavelength of red cadmium light, so that even if the standard meter bar were destroyed, another could be constructed by reference to the wavelength of red cadmium light. During 1946 it became possible through advances in atomic physics to prepare samples of special materials that have even more desirable characteristics than cadmium for this purpose. The standard of length probably will be based utlimately on one of these new materials (e.g., an isolated isotope of mercury about which the student need not be concerned just yet). It is worth remarking that, by certain rather complicated optical methods, distances can be measured to within a millionth of an inch. Such methods are discussed in advanced courses.

In steps 1 - 5 in the Procedure of this experiment the dimensions of an aluminum block and the diameter of a steel ball are measured with an ordinary scale, a vernier caliper, and a micrometer caliper. The methods of using these instruments are discussed in the section of the laboratory manual on Measuring Instruments.

B. The mass of a body is sometimes defined as being the quantity of matter in the body. The mass is determined by the inertia of the body, which is related to the "resistance" the body offers to being acceler-

ated (for instance, to being set into motion or to being shaken back and forth).

The fundamental unit of mass is the kilogram, originally intended to be the mass of 1000 cm$^3$ of water at its maximum density. As it happened, the original determination of the density of water was in error by a small fraction of one one-hundredth of a percent, so that the standard of mass for the entire world is the mass of a block of platinum-iridium called the standard kilogram, kept in a vault near Paris. The kilogram is, of course, 1000 grams.

All civilized countries, except English-speaking ones, have adopted the metric system. The yard is defined as $\frac{3600}{3937}$ meter, and the pound is defined as 0.45359 kilogram

In steps 6 - 9 of the Procedure of this experiment, the mass of an aluminum block and the mass of a steel ball are determined by comparing them with standard masses, using a trip scale and also using an analytical balance. In addition, a spring scale is used, although this measurement is open to some question. For this experiment it will be best to ignore the logical difficulty that the spring scale really measures the pull of gravity on the block rather than its mass. The methods of using these instruments are discussed in the section of the laboratory manual on Measuring Instruments.

C. The fundamental unit of time is the second, which is defined as being $\frac{1}{86,400}$ part of a mean solar day; the latter in turn has been found to be $\frac{1}{365.24219}$ part of a tropical year--something astronomers know how to measure. This "high-brow" definition of a second need not prevent one from recognizing that a second is the interval of time required for the second hand of a clock to move a distance corresponding to a second.

In steps 10 - 12 of the Procedure of this experiment, the time interval between successive swings of a pendulum is determined by means of a clock and also by means of a stop watch.

D. Ordinarily, when a physical quantity is measured several times, the results do not agree precisely. In measuring a length, however, the results of successive measurements occasionally seem to be identical, a circumstance associated with the simplicity of the measurement. In step 13 of the Procedure of this experiment the distance between scratches on the heads of two nails about 395 cm apart in a board is to be measured with some special rulers, which have been designed purposely to be as awkward to use as possible. On account of the awkwardness no one measurement is very accurate, and successive measurements do not agree precisely with each other, but it is found that the average of several measurements is ordinarily more accurate than any one reading.

E. If the student does not already know the meaning of such prefixes as centi-, milli-, kilo-, etc., he should

look them up in a dictionary and write their meanings in the table below. In science it is customary to use the Latin prefixes for fractional parts (thus one centimeter is one one-hundredth part of a meter), and Greek prefixes for multiples (thus a kilometer--accent on the first syllable--is one thousand meters).

| Prefix | Greek or Latin | Meaning | Example |
|--------|----------------|---------|---------|
| mega- | _____ | _____ | _____ |
| kilo- | _____ | _____ | _____ |
| deci- | _____ | _____ | _____ |
| centi- | _____ | _____ | _____ |
| milli- | _____ | _____ | _____ |
| micro- | _____ | _____ | _____ |

## PROCEDURE

Note: Unlike most experiments, there are 13 parts to this one. The instructor probably will designate certain steps to be omitted. It is not necessary to do all steps in sequence, and in order to prevent confusion, it is suggested that some members of the class begin on step 1, others on step 6, others on step 10, and others on step 13.

1. Measure the thickness of the aluminum block three times with the ordinary centimeter scale, placing it on edge against the block to eliminate the error of parallax (see discussion in the section on Measuring Instruments). Compute the average.

2. Measure the thickness in three different places of the aluminum block with the vernier caliper, resetting the caliper each time. Remember to check the zero setting of the caliper. If there is a zero error, record the corrected thickness in the space on the Data Sheet.

3. Measure the thickness of the aluminum block in three different places with the micrometer caliper, resetting the caliper each time. Remember to check the zero setting of the micrometer caliper. Compute the average thickness. (You may wish to practice first on the brass blocks with the thickness stamped on them. There are eight thicknesses of blocks--you may trade with your fellow students if you like.)

4. Measure all the other dimensions of the aluminum block at least three times with the vernier caliper. Compute the averages.

5. Measure the diameter of the steel ball at least three times with the micrometer. (Caution.) Compute the average.

Note: Spend not more than 45 minutes on steps 1-5.

6. Place the aluminum block on the spring scale and make the reading. Jiggle the block to make sure the spring isn't sticking, and repeat.

7. Determine the mass of the aluminum block by

comparing it with the (inexpensive) laboratory standards on the trip scale. After you have obtained the best balance you think possible, move the slider on the scale a few tenths of a gram and note if you then think the balance is as good as before. You get a measure of the precision of your measurement by noting the greatest amount you can move the slider without significantly altering the balance.

8. Determine the mass of the aluminum block by comparing it with the laboratory analytical standards on the analytical balance. (Caution.) Although the analytical balance can be used with care to a precision of 0.1 milligram, it will be sufficient for this experiment to make the determination to the nearest milligram. You can save time by using the method of swings and interpolating as discussed in the section on Measuring Instruments. Remember to raise the balance from its pivots when you are finished.

9. Determine the mass of the steel ball on the analytical balance. (Caution.) See step 8.

Note: Spend not more than 45 minutes on steps 6 - 9.

10. Set the pendulum swinging gently. Looking at the second hand of a clock, determine the period of the pendulum, i.e., the time for the pendulum to swing from the bottom of its swing to one side, back to the other side, and return to the bottom in the original direction. Repeat the measurement a few times and compute the average period.

11. Repeat step 10 using the stop watch. (Caution.)

12. Repeat step 10 using either clock or watch, but instead of timing a single swing, time ten or twenty swings. Repeat the measurement, then compute the average and the period.

Note: Spend not more than 20 minutes on steps 10 - 12.

13. Measure the distance between the scratches on the two nail heads in the special board having the nails about 395 cm apart using the awkward 10-cm rulers. To hold your place after you set one end of the ruler on the center of the nail head, you may use a toothpick, matchstick, hairpin, or even your pencil if you do not mark the board too much. If the entire class proceeds with the measurement in the same direction, several students may work simultaneously. For the sake of the other students, please do not dilly-dally. You are requested to work at a reasonably rapid pace. Your instructor realizes that the time spent on this step is not likely to be inspirational or practical, but by doing the experiment you are cetrain to be impresse by the fact that because of accidental or random errors successive measurements of the same quantity do not all agree with each other, and that the average of several readings is likely to be more accurate than any one reading. Measure the length ten times, but do not spend more than 20 minutes doing it. Compute the average of your measurements, and then measure the distance with the special 4-meter stick.

## DATA AND CALCULATIONS

a. Carry out any computations indicated on the Data Sheet. Be sure to give the proper units for all quantities. For part 13, after you determine your average value for the distance between the two nail heads, compute the deviation of each reading from the average and enter the results in the appropriate blanks. Then compute the average of your deviations (regarding all signs as positive), thereby obtaining the average deviation of your readings from your average. From this value determine the probable error of your result, or if you prefer, the probable average deviation of your result (see the section of the laboratory manual on Theory of Errors), and then enter your final result for the distance in the blank space provided. State whether you used probable error or probable average deviation.

b. From the dimensions of the aluminum block, calculate its volume. From the volume of the block and its mass compute its density. (Density is mass per unit volume, e.g., kilograms per cubic meter. In this case it is convenient to measure density in grams per $cm^3$.) Look up the accepted value for the density of aluminum either in the text or in a handbook such as the Handbook of Chemistry and Physics, and give the percent discrepancy between your value and the handbook value, regarding the handbook value as the proper value. Repeat for the steel ball. Remember to give the proper units.

c. (Optional at discretion of instructor.) Determine an estimate for the probable errors (or probable average deviations) for all the dimensions of the aluminum block. Then combine these probable errors so as to determine the probable error in your result for the density.

## DISCUSSION

On a separate sheet clipped (not pinned) to the Data Sheets of this experiment, type, print, or write legibly in ink not to exceed one page in answer to these questions.

1,2,3,4. Discuss such things as basic ideas illustrated by the experiment, conclusions to be drawn from it, quantitative estimates of sources of error, and possible improvements in the experiment.

5. What are the relative advantages of the various instruments?

6. Does the accuracy of the measurement of a quantity increase if one makes several readings? Why?

7. Why does your result for the density of aluminum not agree exactly with the handbook value?

8. With increasing temperature aluminum expands more than steel. On a cold day does the aluminum block measure longer or shorter than on a hot day?

9. Did your measurement with the 4-meter stick fall inside or outside the probable error for the measurements with the awkward rulers?

10. What is a micron? What is a microfarad? If a radar frequency is 3000 $\frac{megacycles}{sec}$, how many cycles does it make per second?

11. What is the volume of a cylinder whose height is 100 ± 3 cm and whose diameter is 50 ± 1 cm? (You may leave the answer in terms of .) What is the volume of a rectangular block whose height is 100 ± 3 cm and whose length and width are both 50 ± 1 cm? Explain why the percentage or fractional probable errors are different in the two cases.

EXPERIMENT 51-2, FUNDAMENTAL MEASUREMENTS　　　　4

DATA SHEET

| Step | | | | | Single reading or average (give units) |
|------|--|--|--|--|----------------|
| 1 | Thickness Al, cm scale | _____ | _____ | _____ | _____ |
| 2 | Zero reading, vernier | _____ | _____ | _____ | _____ |
|   | Thickness Al, vernier | _____ | _____ | _____ | _____ |
| 3 | Zero reading, micrometer | _____ | _____ | _____ | _____ |
|   | Thickness Al, micrometer | _____ | _____ | _____ | _____ |
| 4 | Length of aluminum block | _____ | _____ | _____ | _____ |
|   | Width of aluminum block | _____ | _____ | _____ | _____ |
|   | Diameter of hole in block | _____ | _____ | _____ | _____ |
| 5 | Diameter of steel ball | _____ | _____ | _____ | _____ |
| 6 | Aluminum on spring scale | _____ | _____ | _____ | _____ |
| 7 | Aluminum on trip scale | | | | _____ |
|   | Maximum "un-noticeable" unbalance | | | | _____ |
| 8 | Rest point, analytical balance (see also "Other Data") | | | | _____ |
|   | Aluminum, analytical balance | | | | _____ |
| 9 | Steel ball, analytical balance | | | | _____ |

Calculations (need not be done in lab)

| | | | |
|--|--|--|--|
| Volume of aluminum block | _____ | of ball | _____ |
| Mass of aluminum block | _____ | of ball | _____ |
| Density of aluminum block | _____ | of ball | _____ |
| Handbook density of aluminum | _____ | of ball | _____ |
| Percent discrepancy for block | _____ | of ball | _____ |

| Step | | | | | |
|------|--|--|--|--|--|
| 10 | Period of pendulum, clock | _____ | _____ | _____ | _____ |
| 11 | Period of pend., stopwatch | _____ | _____ | _____ | _____ |
| 12 | Time for _____ periods | _____ | _____ | _____ | _____ |
|    | Period computed from line above | | | | |

EXPERIMENT 51-2, FUNDAMENTAL MEASUREMENTS          5

DATA SHEET (Continued)

Step

Deviation
from
average

13  Distance between nail heads  _____                    _____

                                _____                    _____

                                _____                    _____

                                _____                    _____

                                _____                    _____

                                _____                    _____

                                _____                    _____

                                _____                    _____

                                _____                    _____

                                ===============                    ===============

           Totals  _____

           Average  _____

                                                      Average
                                                      Deviation  _____

                                        Probable Average Deviation  _____

Final Result (give units and Prob. Av. Dev.)        _____

Measurement with special 4-meter stick              _____

Other data:
    (For example, some students may wish to record the calibration corrections of the precision
    standards used on the analytical balance.)

Approval_____

Hours spent on this experiment: Study_____Lab_____Report_____

EXPERIMENT 51-3
ACCELERATION OF GRAVITY

## OBJECT

To investigate the motion of a freely falling body, to verify that the acceleration of gravity is uniform, and to determine the magnitude of the acceleration of gravity.

## APPARATUS

In Lab: Free-fall apparatus, iron bob, uniform time-interval sparking device set for 30 sparks per second, meter stick, two-meter stick.

Optional, check out: Stop watch, revolution counter.

(Note to Lab assistants: The spark-control motor should be plugged into the 117-volt AC line near station 6 in room 394. The switch in the AC distribution line around the room should be opened near station 12 so that this wire--which should be black and was, before the painters painted it white--can be used to distribute spark impulses. Plug the DC inlet into the 130-volt DC line near station 8, being careful to get the proper polarity (i.e., hot or ground). Connect the hot side of the output to the spark primaries to the previously opened AC line. Close the spark primary switches at the individual stations and you should get sparks. A complete circuit diagram of the spark motor should be kept with the motor.)

## THEORY AND METHOD

A. In this experiment an iron bob is held about eight feet above the floor by an electromagnet. When the current in the electromagnet is turned off, the bob falls freely to the floor with only air resistance to hinder its motion. This resisting force is negligible for such a heavy object falling such a short distance, so that we may regard the pull of gravity as being the only force acting on the bob which is, therefore, accelerated toward the earth.

In the present experiment, see Figure 1, the iron bob falls between two parallel vertical wires, over one of which is placed a strip of waxed paper. By means of a commutator driven by a synchronous motor running at 1800 rev per min, and an electrical spark coil, a high voltage is connected to the wires every one-thirtieth of a second. An electric spark jumps from one of the wires to the bob to the other wire, leaving a small hole burned in the waxed paper where the spark has passed through. By measuring the position of the holes, the distance the bob has fallen in any particular thirtieth of a second can be determined. The time interval between sparks is determined by the same 60-cycle per sec power-line frequency that operates electric clocks. Its accuracy is ordinarily better than one part in 10,000.

B. Anticipating one of the results of the experiment, let us assume the acceleration of gravity g is uniform. It is shown in lecture that the distance s, through which the bob falls from rest in a time t, is:

$$s = \frac{1}{2} gt^2 \qquad * \quad (1)$$

By measuring the distance the bob falls in various times, we might hope to verify Equation (1) and to determine the value of g from the data and Equation (1). There is, however, one difficulty with determining g from Equation (1). It is that there is no satisfactory way of releasing the bob precisely at the time when a spark jumps. This difficulty can be circumvented by the method described below.

C. Let a body have an initial velocity $v_0$, let it have a uniform acceleration a during a period of time t, and let its final velocity at time t be v.

It is shown in lecture that

$$v = v_0 + at. \qquad * \quad (2)$$

The existence of a linear relation between v and t implies that the acceleration is a constant. In the present experiment the acceleration with which we are concerned is the acceleration of gravity g. Accordingly, if the acceleration of gravity is a constant, then Equation (2) becomes

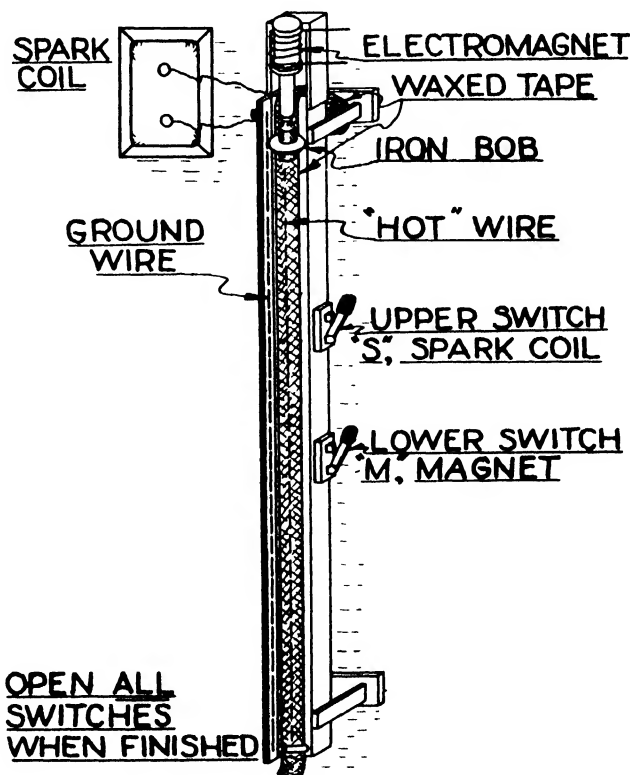$$v = v_0 + gt. \qquad * \quad (3)$$



Figure 1. Free-Fall Apparatus

The student should review the section of the laboratory manual on Graphs.

If we plot a graph of the velocity of the freely falling iron bob as a function of time, and if we find it to be a straight line, then we know the acceleration of gravity is a constant (not a variable) and that the slope of the straight-line graph is g. We thereby accomplish the objective of the experiment.

Note: Usage varies with regard to signs for g. Clearly, the acceleration of gravity is downward. Accordingly, some people, including the author, use Equation (3) and regard g as being a negative number, e.g., $-9.8 \frac{m}{sec^2}$. Others prefer to regard g as a positive number, and write $v = v_0 - gt$. Still others prefer to regard the downward direction as a positive direction. The choice is a personal matter.

D. Actually, we do not measure the velocity directly. We measure the distance between marks on the waxed tape and the time intervals. The ratio of the distance the bob falls in any time interval to the length of the time interval is the average velocity of the bob during the interval. We may take the average velocity of the bob during any time interval to be its instantaneous velocity in the middle of the time interval, thus: if the distance at the end of the first one-thirtieth of a second is 0.0055 m and the distance at the end of the second one-thirtieth of a second is 0.0218 m, then the average velocity during that thirtieth of a second is

$$\frac{0.0218 - 0.0055}{\frac{1}{30}} = (0.162 \times 30) = 0.489 \frac{m}{sec}.$$

We may take the instantaneous velocity at a time of 1.5 thirtieths of a second as being $0.489 \frac{m}{sec}$. In this way we plot a graph of the velocity of the bob as a function of the time. We then determine g from the slope of this line.

E. In addition to the graphical method for determining the acceleration of gravity from the slope of the velocity "curve," there are several possible analytical methods that can be used.

Suppose we imagine the bob to be at a particular point which we shall call the zero point at the time $t = 0$. At this time it may have a velocity $v_0$. It is shown in lecture that for a freely falling body, the distance s traveled in time t is

$$s = v_0 t + \frac{1}{2} g t^2. \qquad * \quad (4)$$

If we represent the time interval between sparks by T; and if we let $d_1$ represent the distance traveled from the zero point in time T, $d_2$ the distance traveled during the second time interval (i.e., $d_1 + d_2$ is equal to the distance traveled from the zero point in time 2T), etc.; and $v_1$ represent the velocity at time T, $v_2$ the velocity at time 2T, etc., then

$$d_1 = v_0 T + \frac{1}{2} g T^2 \quad \text{and} \quad d_2 = v_1 T + \frac{1}{2} g T^2 .$$

From this it follows that $d_2 - d_1 = (v_1 - v_0)T$. But as can be seen from Equation (3), $v_1 - v_0 = gT$, so that $d_2 - d_1 = gT^2$, and hence

$$g = \frac{(d_2 - d_1)}{T^2} .$$

Similarly, one may deduce that

$$g = \frac{(d_3 - d_2)}{T^2} ,$$

and likewise

$$g = \frac{(d_4 - d_3)}{T^2} , \quad \text{etc.}$$

If one now averages the three results for g, he gets

$$g = \frac{[(d_2 - d_1) + (d_3 - d_2) + (d_4 - d_3)]}{3T^2} \qquad (5)$$

It is quite obvious, however, that $d_2$ and $d_3$ cancel out, so that in fact they might just as well not have been measured. If more than four distances are used, it turns out that in general only the first distance and the last distance measured have any influence on the result. Clearly, this is a very foolish method of averaging the results, because most of the work is wasted. The purpose of mentioning the matter in this experiment is to emphasize that after one gets experimental data, he must use some sense in what he does with them.

F. Although the graphical method discussed in sections C and D is a satisfactory method, the most accurate method of reducing the data (as the expression goes) to determine g, is the method of least squares. The method of least squares rests on a firm experimental and theoretical foundation, although the theory is too complicated to be discussed here. It is discussed in books on higher mathematics, numerical methods, and statistics.* The name comes from the fact that the sum of the squares of the deviations from the best value is made a minimum. The result applicable to the present case is merely stated below.** After the student has determined g graphically, he should substitute his data into one of the following formulae to find the best value for g.

Let $s_1$ represent the distance traveled from any arbitrary point (presumably the zero point) in time T, $s_2$ represent the distance traveled from the point in time 2T, etc. If six distances have been measured, then

$$a = \frac{[5(s_1 + s_6) - (s_2 + s_5) - 4(s_3 + s_4)]}{28T^2}. \qquad (6)$$

---

* For instance, see Worthing and Geffner, Treatment of Experimental Data (Wiley).

** See Pugh, Acceleration Calculations from Spark Recorded Data, American Physics Teacher, volume 4, page 70, May 1936.

If ten distances have been measured, then

$$a = [6(s_1 + s_{10}) + 2(s_2 + s_9) - (s_3 + s_8) - 3(s_4 + s_7) - 4(s_5 + s_6)]/132T^2 \qquad (7)$$

If fourteen distances have been measured, then

$$a = [13(s_1 + s_{14}) + 7(s_2 + s_{13}) + 2(s_3 + s_{12}) - 2(s_4 + s_{11}) - 5(s_5 + s_{10}) - 7(s_6 + s_9) - 8(s_7 + s_8)]/728T^2 \qquad (8)$$

7. The value of g is not the same everywhere in the world, varying by more than half a percent—an amount considerably greater than the accuracy with which world's records in track and field events are specified. In a place like Singapore the shot-put could be expected to go half a foot farther than in Norway if handled the same way, merely because g is smaller at Singapore than at Norway.

Practical utility is made of the variation of gravity with position on the earth's surface by oil-prospecting companies. Surveryors determine elevations accurately over a course. Gravity varies with elevation in a well-known manner (inversely as the square of the distance from the center of the earth), and if the gravity-meter determinations do not yield the same elevations, the presence of oil (often located in salt domes) may be indicated. Gravity meters are sensitive enough to measure differences in elevation of less than a foot. Naturally they are more involved than our simple free-fall apparatus.

## PROCEDURE

1. The apparatus will be found with the electrical connections already made. Open and close only those switches referred to in these directions.

2. Thread the waxed paper, white side out, over the top of the free-fall apparatus. Center the paper relative to the wire and clamp the upper end. Draw the bottom of the paper down, center it, and clamp it tightly.

3. Close the lower switch M, which controls the electromagnet on the free-fall apparatus and suspend the bob motionless (not wobbling) from the electromagnet.

4. With the motor of the sparking device running, close the upper switch S, which controls the spark coil; and open switch M, releasing the bob and permitting it to drop into the cushioned box. Open switch S immediately and leave switch M open also.

CAUTION: While the switch S is closed, do not touch the wires running from the spark coil to the free-fall apparatus or the wire behind the waxed tape. Most high-voltage shocks are unpleasant. Some are dangerous.

5. Unless otherwise directed, each student should obtain a waxed tape record.

6. (Optional) Verify the speed of revolution of the motor a few times, using a revolution counter and stop watch. Measure the number of revolutions in one minute, or the time for 1800 revolutions.

7. (Optional) If a second bob of different size or weight is available, make a run with it.

8. Before you leave the Lab, see that all extra tape, scratch paper, and so forth, are in the waste basket.

## DATA AND CALCULATIONS

a. Remove the waxed tape from the apparatus and stretch it out on the table. You may wish to draw a small circle around each dot to keep it from getting "lost." Omitting the first few holes, draw a line perpendicular to the length of the tape through each hole. Number the dots in succession 0, 1, 2, 3, etc.

b. With a two-meter stick placed on edge to avoid parallax (i.e., by getting the scale in contact with the tape) measure the distance from the point marked 0 to each of the other points in succession. The distance from 0 to the first point is called $s_1$, the distance from 0 to the second point, $s_2$, etc. Avoid using the end of the meter stick. If the points are not all in a straight line on the tape, measure to the lines perpendicular to the length of the tape. Record all the distances on the data sheet. In the first column record the numbers of the successive points.

c. In the second column, opposite the number of each point, record the time, reckoning from the time the point marked 0 was recorded. Since the time between sparks was one-thirtieth of a second, the number of thirtieths of a second for any point corresponds to the number of that point. On the lines left between the successive numbered points, indicate the time elapsed to the middle of that interval, e.g., 1/60, 3/60, 5/60 sec., etc.

In the third column record the distance s from the zero point 0, to the point in question.

Calculate the distance between successive points and enter the results in the fourth column on appropriate lines as shown in the example, entered on the page preceding the Data Sheet.

Since the values tabulated in the fourth column are the instantaneous velocities in meters per thirtieth of a second, the corresponding values of velocity in $\frac{m}{sec}$ may be found by multiplying the entries in the fourth column by 30. Compute the values of the instantaneous velocity in $\frac{m}{sec}$ and tabulate them in the sixth column.

d. Entries in the fourth column give velocities in terms of thirtieths of a second. The difference in or change of velocity per unit time is the acceleration. Take differences of the fourth column and enter them in the fifth column. Since these accelerations are meters per thirtieth of a second per thirtieth of a second, the acceleration in $\frac{m}{sec^2}$ may be found by multiplying the entries in the fifth column by 30 x 30 = 900. Enter the results for the acceleration in $\frac{m}{sec^2}$ in the seventh column. One might think

that averaging all these results for the acceleration would give a reliable result, but actually it can be shown that the averaging of the entries in column seven is determined only by the first two and by the last two entries in column three, all the others being "wasted." This is another example of an unsatisfactory method of averaging.

e. Using engineering cross-section paper having 20 lines per inch, plot a graph of the velocity in

$\frac{m}{sec}$ as a function of time. Choose the time as the independent variable and plot it as abscissae (horizontally); and the velocity as the dependent variable, and plot it as ordinates (vertically). Indicate each point by a small cross. Draw one straight line as near to most of the points as possible with the points distributed fairly evenly on both sides of the line all along the line. Determine the slope by using one point near either end of the line. The points used for determining the slope should be points on the line, not experimental points. Review the section on Graphs marked Rules for Drawing Graphs.

f. Having determined the slope of this experimental line, make some estimate of the uncertainty in its slope by placing a straight edge on the graph and noting the deviation in slope possible without getting most of the points at one end of the line on one side of it, and most of the points at the other end on the other side. Indicate this limit of uncertainty by the symbol ± in your final result for g calculated from the slope. Compare your result for g with the value obtained by the U.S. Coast and Geodetic Survey.

g. Note: Since this experiment takes less than one second to perform, the student should direct his thinking toward proper presentation of results, both numerically and graphically.

## DISCUSSION

1, 2, 3, 4. On a separate sheet clipped to the sheets of this experiment, write not to exceed one page, discussing such things as the basic ideas illustrated by the experiment, conclusions to be drawn from it, quantitative estimates of sources of error, and possible improvements in the experiment.

5. The fact that the "curve" of the velocity as a function of the time is a straight line proves what?

6. In section D of the Theory and Method we took note of the fact that the average velocity in any time interval was the instantaneous velocity in the middle of the time interval. Is this the same as the instantaneous velocity halfway between the two distance points? Explain.

7. Extend your experimental line backward until it cuts both coordinate axes. What is the significance of the intercept of your straight line with the "v" axis? What is the significance of the intercept with the "t" axis?

8. Would it be logical to measure the number of revolutions per minute with an electric clock instead of with a stop watch? If you do not get exactly 1800 revolutions per minute for the motor, to what is the discrepancy probably due?

9. The radius of the earth is about 4,000 miles. Take this distance as $2.0 \times 10^7$ feet. Show that the variation in gravity for an increase of 1000 feet in elevation is about 0.01 percent. If you can't make this calculation easily, using simple algebra and arithmetic, ask your instructor for assistance at the next lab period.

10. Arbitrarily change the reading $s_4$ by adding 0.01 m to it. Show that the average of entries in column seven is unaffected. Comment on the result.

11. What can be proved by step 7 in the Procedure?

12. Do all bodies fall with the same speed? The same acceleration? Explain.

## SAMPLE DATA SHEET

| Number of Point | Time (sec) | Distance from zero point (m) | First Difference (m) | Second Difference (m) | Velocity (m sec) | Acceleration (m sec²) |
|---|---|---|---|---|---|---|
| 0 | 0 | 0.00 | | | | |
| | 1 60 | | 0.0055 | | 0.165 | |
| 1 | 1 30 | 0.0055 | | 0.0108 | | 9.72 |
| | 3 60 | | 0.0163 | | 0.489 | |
| 2 | 2 30 | 0.0218 | | | | |
| | 5 60 | | | | | |

Name_____Box No._____Date in Lab_____

Partner_____Lab Section_____Instructor_____

## EXPERIMENT 51-3, ACCELERATION OF GRAVITY

### DATA SHEET
(Add additional sheet if desired)

| Number of Point | Time (sec) | Distance from zero point (m) | First Difference (m) | Second Difference (m) | Velocity (m sec) | Acceleration (m sec$^2$) |
|---|---|---|---|---|---|---|
| _____ | _____ | _____ | | | | |
| | _____ | | _____ | | _____ | |
| _____ | _____ | _____ | | _____ | | _____ |
| | _____ | | _____ | | _____ | |
| _____ | _____ | _____ | _____ | | _____ | _____ |
| | _____ | | _____ | | _____ | |
| _____ | _____ | _____ | _____ | | _____ | _____ |
| | _____ | | _____ | | _____ | |
| _____ | _____ | | _____ | | _____ | _____ |
| | _____ | | | | _____ | |
| _____ | _____ | _____ | | _____ | | _____ |
| | _____ | | _____ | | _____ | |
| _____ | _____ | _____ | _____ | | _____ | _____ |
| | _____ | | _____ | | _____ | |
| _____ | _____ | _____ | _____ | | _____ | _____ |
| | _____ | | _____ | | _____ | |
| _____ | _____ | _____ | _____ | | _____ | _____ |
| | _____ | | _____ | | _____ | |
| _____ | _____ | _____ | | _____ | | _____ |
| | _____ | | _____ | | _____ | |
| _____ | _____ | _____ | | | | |

t intercept_____ v intercept_____

Motor, time for 1800 rev_____ _____ _____ _____

g (graph)_____ g (least sq.)_____ g (column 7)_____

g (Stanford University, elevation 75 feet, USCGS)____9.7999 m sec$^2$____

Percent error in g (graph)_____least sq._____

Approval_____(Is the lab picked up and neat?)

Hours spent on this experiment: Study_____Lab_____Report_____

EXPERIMENT 51-4
CENTRIPETAL FORCE

## OBJECT

To test the law of centripetal force for a body moving in a circle with uniform speed.

## APPARATUS

In Lab: centripetal-force apparatus ($60), clock, horizontal support and hook, kilogram weight, weight hanger.

Check out: vernier caliper, slotted weights.

## THEORY AND METHOD

A. Newton's second law of motion states that the force required to give a mass m an acceleration a is

$$Force = ma. \qquad * \quad (1)$$

It is shown in lecture and also in the text that for a particle moving in a circular path of radius r at constant speed v, the acceleration is directed toward the center and has the magnitude

$$a = v^2/r \qquad * \quad (2)$$

so that the centripetal force F required to keep a mass m moving in a circular path of radius r at a constant speed v, is directed toward the center of the circle and has the magnitude

$$F = mv^2/r. \qquad * \quad (3)$$

If the angular velocity (in radians/second) of the revolving mass is $\omega$, Equation (3) becomes

$$F = mr\omega^2. \qquad * \quad (4)$$

But

$$\omega = 2\pi f = 2\pi N/t \qquad * \quad (5)$$

where f is the frequency or number of revolutions per unit time, and N is the number of revolutions in the time t. Accordingly, Equation (4) becomes

$$F = 4\pi^2 mrN^2/t^2. \qquad (6)$$

Equation (6) is in a form convenient for experimental test; the force, the mass, the radius, the number of revolutions, and the time all can be determined easily. If agreement is found between the measured and calculated values of the force F, then the basic ideas expressed by Equations (1) to (3) may be regarded as verified.

B. An apparatus similar to that shown in Figure 1 is used to test Equation (6). The frame containing the mass m to be revolved, is driven by an electric motor through a wheel W in a friction transmission; the speed of the transmission can be varied continuously over a wide range by screw A. The necessary centripetal force is provided by a spring S, fastened at one end to

the mass m, and at the other end to an adjustable collar J, which can be set so as to secure different tensions in the spring. The position of the mass can be determined by means of the pointer P, against one end of which the mass rests. If the mass moves outward from the center of the apparatus, the pointer will be pushed upward.
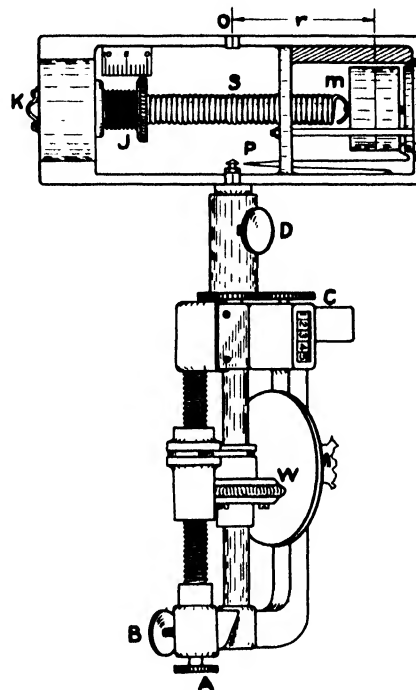


Figure 1. Centripetal Force Apparatus

C. The number of revolutions is determined by a revolution counter connected to the rotating shaft; the time is determined by a clock with a second hand; the radius to the center of gravity of the block is determined with a caliper.

## PROCEDURE

1. Set up the centripetal force apparatus, making sure the frame containing the mass m and the spring S is securely fastened to the rotating mechanism by means of screw D. Engage the friction transmission. Record the number of your apparatus.

2. Set the tension-adjusting collar J near one end of its scale and record the scale reading.

3. Start the motor, and using screw A, adjust the speed of rotation to a slow speed; then increase the speed gradually until the pointer P lifts up and comes level with the marker, as shown in Figure 1. Even though the frame is rotating, it is possible to see the pointer when the eye is held in the same horizontal plane with it. This adjustment is important, and also is difficult. You may find it convenient to maintain the proper speed by applying friction to the shaft above D

by pressing lightly with a pencil or with your finger (caution!), or by looping a strip of paper around the shaft and pulling gently to produce a slight braking action.

If you cannot get a good balance, adjust the speed as nearly as you can to a balance, and then allow the pointer just barely to go up (or down). Make the entire run with the pointer in the up (or down) position. Then repeat, but with the pointer in the down (or up) position. Run additional checks for both runs. You may be certain that the balancing speed is intermediate between the speeds for the up and down positions. If the two speeds are nearly equal, you are certain that you have data whose errors are no greater than indicated by the difference between the up and down speeds. The average speed or mean of all your runs is likely to be quite accurate. You can make an estimate of its accuracy by determining the probable error of the mean.

At the very worst you may allow the pointer to oscillate between the up and down positions, but this is not an accurate method.

4. When you have the speed in good adjustment, record the reading of the revolution counter, and engage it at some precisely noted instant. At some convenient time later (say, after an interval of 100 seconds) disengage the revolution counter, and immediately stop the gear on it from coasting by touching it with the rubber-covered "stopping lever." (Caution!) Record the number of revolutions and the time interval.

During this timing operation the speed of rotation must be kept constantly in balance. Do not expect to get a perfect record the first time. Practice all the operations several times.

Make as many determinations of the critical speed as are necessary in order to give you what you regard as a satisfactorily small probable error. Use your best judgment on the matter, but show your calculations in your report.

5. Either remove the frame and find the value of the force by suspending the frame from a horizontal bar by means of hook K, or readjust the rotator so that the axis of spin is horizontal instead of vertical, and the frame is in a position such that hook K is downward. In either case, connect a weight hanger to the mass m and add weights until the pointer P comes opposite the index mark.

6. While the apparatus is in the adjustment of step 5, with the vernier caliper measure accurately the radius of revolution r, from the center of gravity of the mass m to the axis of revolution O. The distance is clearly indicated by grooves on the apparatus.

7. Repeat steps 1-6 with the tension-adjusting collar J at the other end of its scale.

8. Disengage the friction transmission.

## DATA AND CALCULATIONS

a. Enter suitable data in the spaces provided, and if necessary add additional explanation.

b. When you are determining the force to stretch the spring, do not forget that the mass m is hanging on the

spring in addition to the weights and hanger. The mass of the slug m is stamped on it. To convert force in kilo grams-weight to newtons, or force in grams-weight to dynes, you may use $g = 9.8 \text{m/sec}^2$ or $g = 980 \text{ cm/sec}^2$, respectively.

c. The material discussed in this paragraph is rather subtle and is of secondary importance; students who find it difficult are advised to take the conclusion in quotation marks on faith and not to do any worrying about the matter. When several runs are being made with identical values of m, r, and t, one obtains several values of N. These values of N would be all alike if it were not for experimental error. "The proper way to calculate the best value of F is to determine the average value of N and to square the average." An alternative (but theoretically unsound) method is to square each value of N, then to calculate each value of F, and then to take the average of these values as the best value of F. The reason why the former method is proper is that it is in N not in $N^2$ that the random experimental errors occur.

d. Determine the experimental value of the probabl( error of the force determined from Equation (6) for a given set of "identical" runs combining the probable errors (if any) of the various factors according to stan( ard procedure. In this experiment the symbol ± may be used to indicate probable error rather than limit of error.

DISCUSSION (Instructor should limit the number of questions assigned so that they may be answered in not more than two pages of discussion.)

1,2,3,4. Discuss such things as the basic ideas illustrated by the experiment, conclusions to be drawn from it, quantitative estimates of sources of error, anc possible improvements in the experiment.

5. In this experiment, two aspects of the object of the experiment have not been investigated. Have all qua titles appearing in Equation (4) been varied? Explain.

6. Not all parts of the revolving mass are at the same distance from the axis about which it is being revolved. We measured the radius to the center of mass and assumed that all the mass might be considered to be concentrated there. By the methods of integral calculus it may be proved that this assumption is valid. In this paragraph you are asked to show by algebra and geometry that the assumption is valid for the present case of a cylindrical mass. First, consider two elements on opposite ends of a diameter of the mass in the plane of revolution and resolve the centripetal force for each of them into components parallel and perpendicular to the radius from the axis of revolution to the center of mass. Then consider the cylindrical mass to be made up of a large number of equally thin slices and show that no error arises from the fact that they are not all at the same distance from the axis.

7. Advanced theory shows that one-third the mass of the spring should be added to the mass of the revolving mass m when it is being revolved. (Procedure, step 4.) It is almost obvious that when the force

needed to stretch the spring is being determined (Procedure, step 5, and Data, part b) half the weight of the spring should be added to the weight of the mass. (All of the weight of the spring stretches the top turn but none of the weight of the spring stretches the bottom turn. The average for all turns is half the weight of the spring.) The mass of the spring is about 16.5 grams. What influence on your results do these corrections have? Be explicit in your report as to how you have actually done your calculations concerning these corrections.

8. Do your calculated and measured forces agree within the probable error? Is this to be expected? Explain briefly.

9. Suppose an error of one percent is made in the time interval for a given run. What error does this introduce in F ? (The cause of the error may be non-uniform personal reaction time between reading the clock and moving the stopwatch knob.)

10. Show by an example for a series of real positive numbers n, that ordinarily the square root of the average value of $n^2$ is greater than the average value of n. Comment on the application of this result to part c of Data and Calculations.

11. In any of your runs was the centripetal force greater than the weight of the mass m ?

12. An airplane diving with a constant speed of 500 ft/sec pulls out of the dive with a radius of curvature of 1000 feet. With what force does a 160-lb pilot push on the seat of his pants at the lowest point in the dive?

Name_____Box No._____Date in Lab_____

Partner_____Lab Section_____Instructor_____

## DATA SHEET

Apparatus No.

Mass stamped on slug  m _____ grams = _____kilograms

Radius of revolution  r  (units ?) _____ _____ _____ _____ _____
     (when you take averages, etc., so indicate)

| Run No. | Tension Collar Setting | Revolution-Counter | | Total Revolu- tions N | Time Interval t sec | Calculated Force (newtons) Equation (6) |
|---------|------------------------|-------------------|----------------|-----------------------|---------------------|------------------------------------------|
|         |                        | Before Run        | After Run      |                       |                     |                                          |
| 1       |                        |                   |                |                       |                     |                                          |
| 2       |                        |                   |                |                       |                     |                                          |
| 3       |                        |                   |                |                       |                     |                                          |
| 4       |                        |                   |                |                       |                     |                                          |
| 5       |                        |                   |                |                       |                     |                                          |
| 6       |                        |                   |                |                       |                     |                                          |
| 7       |                        |                   |                |                       |                     |                                          |
| 8       |                        |                   |                |                       |                     |                                          |
| 9       |                        |                   |                |                       |                     |                                          |
| 10      |                        |                   |                |                       |                     |                                          |
| 11      |                        |                   |                |                       |                     |                                          |
| 12      |                        |                   |                |                       |                     |                                          |

|                    | Weights Added | Measured Force (newtons) | Calculated Force - copy from above | Difference in percent |
|--------------------|---------------|--------------------------|-------------------------------------|-----------------------|
| Runs_____ to _____ | _____ | _____ | _____ | _____ |
| Runs _____ to _____ | _____ | _____ | _____ | _____ |
| Runs _____ to _____ | _____ | _____ | _____ | _____ |

Other data or remarks:

Approval _____ (Instructor will check on step 8 of Procedure)

Hours spent on this experiment:  Study_____Lab_____Report_____

## EXPERIMENT 51 - 5
## MOMENTUM OF A RIFLE BULLET

### OBJECT

To test the law of conservation of momentum.

### APPARATUS

In Lab: 22 rifle, clamped to table; whirling cardboard disks; scotch tape; revolution counter; clock; meter stick; protractor for disks; ballistic pendulum, properly mounted (see Theory and Method, G, below); marker; trip scale and weights; analytical balance and weights.

Obtain cartridges and a clean rag from the instructor.

### THEORY AND METHOD

A. In this experiment the velocity of a rifle bullet is measured directly by firing it through two cardboard disks in a direction parallel to the shaft on which the two disks are mounted about 8 feet apart; see Figure 1. The shaft is rotating at a known rate and whirls the disks around. The bullet goes through both disks, and the angle through which the disks have turne while the bullet is traveling the distance between them gives a measure of its velocity.

After passing through the second disk, the bullet enters a ballistic pendulum and stops in the pendulum bob. From the known masses of the bullet and pendulum, and the swing of the pendulum, one may compute the original velocity of the bullet using the principle of conservation of momentum. If agreement is found between the measured and calculated velocities of the bullet, the principle may be regarded as verified.

It is hardly necessary to point out that the ballistic pendulum·finds considerable use in ordnance laboratories.

B. The (linear) momentum of any object is the product of its mass and velocity. In case two bodies of mass $m$ and $M$, whose velocities are respectively $u_1$ and $u_2$, collide with each other, and whose velocities after impact are respectively $v_1$ and $v_2$, the principle of conservation of momentum states that

$$mu_1 + Mu_2 = mv_1 + Mv_2 \qquad * \quad (1)$$

In other words: the sum of the momenta before impact equals the sum of the momenta after impact.

The bullet remains in the pendulum, so that after the collision the bullet and the pendulum have the same velocity, hereafter referred to as $v$. Since the pendulum is at rest before the impact, its velocity at that time is zero. Hence, if we use $m$ for the bullet, and $M$ for the pendulum, Equation (1) becomes

$$mu_1 = (m + M)v. \qquad (2)$$

Equation (2) contains four quantities. We may determine the two masses by using suitable scales. If we could determine the velocity after impact $v$, we could then use Equation (2) to calculate the original velocity of the bullet.
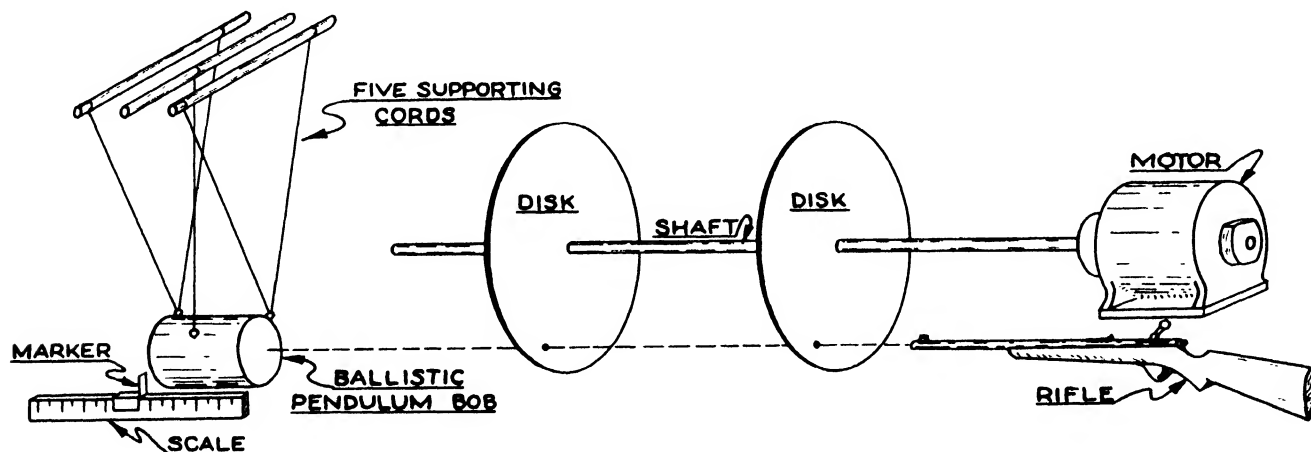


Figure 1. Rifle Bullet Apparatus. Note how all hooks of the ballistic pendulum are on the same level if all cords are tied to the support at the same level.

C. We cannot measure v directly, but if we make the reasonable assumption that the energy lost because of air friction while the pendulum is swinging is negligible (at least for one swing), we can make a measurement that enables us to calculate v. Specifically, if the pendulum swings upward a maximum height h, its kinetic energy at the bottom of its swing is equal to its potential energy at the top, thus

$$\tfrac{1}{2}(m + M)v^2 = (m + M)gh, \qquad * \quad (3)$$

where g is the acceleration of gravity. Equation (3) is easily reduced to

$$v^2 = 2gh. \qquad * \quad (4)$$

D. The height to which the pendulum swings is not great, however, and is not easily measured. It is simpler and more accurate to measure the maximum length of the swing x, and to compute the height from a theorem in geometry known as the "sagitta formula."

Referring to Figure 2, it is easily seen from the Pythagorean theorem that

$$(R - h)^2 + x^2 = R^2 \qquad (5)$$

where R is the radius of the arc in which the pendulum swings. Therefore

$$R - h = \sqrt{R^2 - x^2}$$

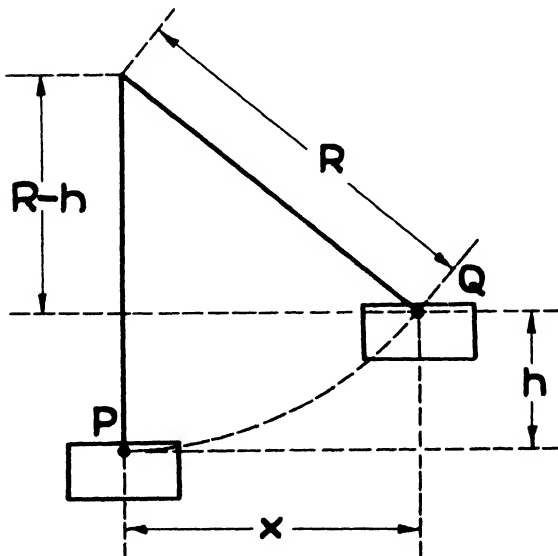or $\qquad\qquad\qquad\qquad\qquad (6)$

$$h = R - \sqrt{R^2 - x^2}$$



Figure 2. Sagitta Relationship

This expression may be expanded by the binomial theorem to yield

$$h = \frac{x^2}{2R} + \frac{x^4}{8R^3} + \frac{x^6}{16R^5} + \ldots \qquad (7)$$

When x is small compared to R, Then only the first term of the right member is required, and we have

$$h \cong \frac{x^2}{2R} \qquad * \quad (8)$$

which usually is referred to as the "sagitta formula."

E. In our experiment the distance x and the radius R are measured, from which h may be computed by Equation (8). This value of h is substituted in Equation (4), which may then be used to calculate v, v being then substituted in Equation (2) to calculate the initial velocity of the rifle bullet. The result of combining Equations (8), (4), and (2) is

$$u_1 = \frac{(m + M)x}{m}\sqrt{\frac{g}{R}}. \qquad (9)$$

F. The initial velocity of the bullet is also measured directly. With the disks whirling, the bullet is fired through the first disk leaving a hole at point A; see Figure 3. The bullet travels through the second disk, leaving a hole at point B. B and A are not in line because the second disk has rotated through an
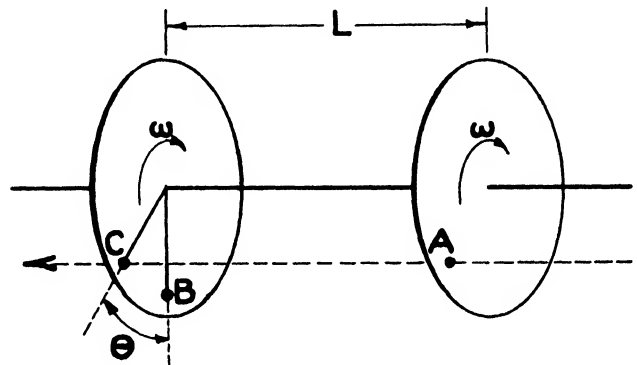


Figure 3. Bullet Path through Disks

angle ϑ during the time the bullet is in flight. To simplify the measurement of the angular displacement of hole B relative to hole A, the disks are stopped, and a second bullet is fired through the original hole A, leaving another hole C, in the second disk. It is a simple matter to measure the angle x between the holes B and C with a special protractor provided for the purpose. Take care to use the proper side of the protractor arm.

If the distance between the disks is L, and the time required for the bullet to travel from one disk to the other is t, then clearly its initial velocity $u_1$ is: $u_1 \quad \frac{L}{t}$. If the disks have an angular velocity and turn through an angle ϑ while the bullet is traveling from one to the other, then likewise,

$\omega = \frac{\theta}{t}$ . These last two expressions may be combined to yield

$$u_1 = \frac{L\omega}{\vartheta} \ . \qquad\qquad (10)$$

L may be measured with a meter stick, $\vartheta$ with a protractor, and $\omega$ by determining the time for a number of revolutions counted with a revolution counter.

G. It is worth while to call attention to the method of suspending the pendulum bob. Every rigid body has six degrees of freedom, that is, when the body is set in motion it may move in any of six independent ways. It has three degrees of translational freedom, corresponding to the motions up and down, north and south, and east and west. It has also three degrees of rotational freedom corresponding to rotations about three mutually perpendicular axes, such as those oriented up and down, north and south, and east and west. In general, the motion will be a combination of these six independent modes of motion. Each of the five cords suspending the bob so constrains it as to remove one of these degrees of freedom. The result is that the bob can move in only one way: it swings in an arc, but in doing so it cannot move sideways, nor can it rotate about any axis. By a little experimentation you can verify that this is true unless the forces used are so large that some of the cords actually become loose. The vertical radius of swing as determined by all lines should be the same. (Instructors please note; see Figure 2.)

The bob is mounted in accordance with the principles of "kinematic design," in which one constraint is used to remove one degree of freedom. The usual three-legged stool does not wobble, as it also is kinematically designed. A four-legged stool, however, has four constraints to remove only three degrees of freedom, so that unless the legs are exactly the correct length, the stool wobbles. Books have been written on kinematic design and designers of precision machinery should be familiar with the subject.

PROCEDURE

1. The rifle will be found already clamped in position. Remove the gunlock without unclamping the rifle and bore-sight the rifle at the ballistic pendulum. If the rifle is not properly aimed, call the instructor, who will supervise matters. Unauthorized unclamping of the rifle, or failure to take all possible precautions in firing the rifle will result in severe disciplinary action against the offender. Any time the rifle is moved it must be bore-sighted again.

2. Adjust the disks in position if necessary, and with scotch tape cover any holes remaining from previous experiments.

3. Optional. Determine the mass of the pendulum bob by carefully lifting the bob without detaching the cords and sliding a trip scale under it.

4. Adjust the ballistic pendulum bob and place a scale with marker in position so as to measure the horizontal swing of the pendulum. A meter stick placed on edge with a small metal slider can be used to measure the swing. Record the position of the slider when it is touching the pendulum bob in its rest position. You may anticipate a swing of about 10 cm horizontally. To help reduce the effect of sliding friction, you may move the slider back about 9 cm.

5. Wipe excess grease from the cartridge with a clean rag; then, on an analytical balance, determine the mass of the cartridge to the nearest milligram.

6. Start the disks rotating and measure the number of revolutions of the shaft for some convenient time, say 100 seconds. For the sake of accuracy you may wish to make this measurement more than once. (Some motors require a warm-up of several minutes before their speed becomes reasonably constant.)

7. All members of the class must stand behind the rifles. The rifles may then be loaded. When all groups are ready, the instructor will give notice to fire, all rifles being fired simultaneously while all members of the class are out of the way. Rifles must never be fired except when the instructor directs.

8. Determine the mass of the shell on the analytical balance. The mass of the bullet is equal to the mass of the cartridge minus the mass of the shell minus the mass of the powder, which may be taken to be $0.122 \pm 0.001$ grams for a long-rifle 22-caliber cartridge.

9. Measure the swing of the pendulum. If the pendulum wobbled badly, it may be wise to repeat most of steps 2 to 8.

10. You may wish to determine the mass of the bob once more. It must be done at least once, either in step 3 or in step 10. Note that sometimes paraffin is knocked out of the bob.

11. Turn off the motor and allow the disks to come to rest slowly so as not to shift the disks on their shaft. Adjust them so that a second bullet may be fired through the hole in the first disk. Make sure the pendulum bob is in position. All members of the class again stand behind the rifles, after which they may be loaded, and fired when the instructor directs.

12. With the special protractor measure the angular separation of the two holes in the second disk. (There are two different kinds of protractors because the whirling disks are not all alike.)

13. Measure the distance L between the disks.

14. Measure the radius of swing of the pendulum. Give special thought to the question of exactly what distance to measure. To measure this distance with a meter stick may take some ingenuity, but it can be done accurately.

15. If there is time, the instructor may direct you to make a second complete run, perhaps with another type of cartridge.

16. Cover all holes in the disks with scotch tape. Hang the ballistic pendulum properly, and arrange the apparatus in an orderly fashion.

## DATA AND CALCULATIONS

a. Take data on the data sheet. Make necessary computations to determine the "calculated" and "observed" velocities of the bullet. Submit your calculations on a separate sheet.

b. It may happen that no energy is lost during a collision, in which case the impact is said to be elastic, but in the present experiment the collision of the bullet with the ballistic pendulum is inelastic. Calculate the kinetic energy of the bullet $\frac{1}{2} mu_1^2$, and the kinetic energy of the pendulum after impact $\frac{1}{2}(m + M)v^2$, and thus determine the fraction of the original kinetic energy that has been lost. Use the observed value for $u_1$.

c. While making computations, take care to keep all your units straight. It is well to keep length in meters, mass in kilograms, and time in seconds. In this case force is in newtons, and energy in joules. Although you may use radians for angular measure when you are determining the angular velocity of the disks, for this particular experiment it is probably simpler to use revolutions for angular measure, and it is suggested that you use revolutions.

d. You will do well to indicate some limit of error for the various individual measurements, e.g., $\vartheta = 45^\circ \pm 0.5^\circ$. You may wish to calculate the resulting limit of error to see if your two measurements of the velocity of the bullet fall within their limits of error.

## DISCUSSION

There is more to this experiment than to many of the others, so your discussion may be somewhat longer than for other experiments. (Instructor should limit the number of questions assigned so that they may be answered in not more than two pages of discussion.)

1;2;3,4. Discuss such things as the basic ideas illustrated by the experiment, the conclusions to be drawn from it, quantitative estimates of sources of error, and possible improvements in the experiment.

5. Suppose R = 100 cm, and x = 10 cm. What percent error is made in calculating h by Equation (8)? What percent error is consequently introduced into the value of $u_1$ ?

6. Show how to obtain Equation (7) from Equation (6).

7. In what part of this experiment is mechanical energy conserved? In what part is it not conserved?

8. Why is the kinetic energy of the pendulum after impact so much less than the kinetic energy of the bullet before impact? What happens to this lost kinetic energy?

9. Show that the loss of kinetic energy during the impact is $\dfrac{M}{m + M}(\frac{1}{2} mu_1^2)$. How does your result for the fraction of energy lost compare with the result of this paragraph?

10. Two separated V-shaped blocks support a single horizontal cylinder. Providing the cylinder remains in contact at its four constraining points, how many degrees of freedom does it have and what are they?
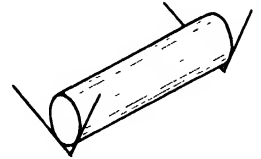
11. Explain the reason for step 9 in the Procedure.

12. From where to where should R be measured (step 14 in Procedure)?

13. How far may a bullet fired horizontally with a speed of 1,000 ft per sec be expected to fall while traveling a horizontal distance of 10 ft? of 100 ft?

14. It is interesting to estimate the change in velocity of the bullet when it goes through a cardboard disk. Assuming that both bullets used in the experiment leave the rifle with equal velocities, the bullet that goes through holes A and B in Figure 3 makes two holes in the disks, while the second bullet finds hole A already made, and makes only the one hole, C. By carefully comparing the swings of the pendulum in the two cases, it is possible to calculate the loss in speed of the bullet in penetrating a disk, at least to the limit of the assumption about both bullets having equal velocities originally.

Name_____ Box No._____ Date in Lab_____

Partner_____ Lab Section_____Instructor_____

DATA SHEET

| | | |
|---|---|---|
| Apparatus number | _____ | _____ |
| Mass of pendulum (with bullet?) | _____ | _____ |
| Type of cartridge | _____ | _____ |
| Mass of cartridge | _____ | _____ |
| Mass of shell | _____ | _____ |
| Mass of bullet | _____ | _____ |
| Position of rider, pendulum at rest | _____ | _____ |
| Position of rider, maximum swing | _____ | _____ |
| Horizontal displacement of pendulum, x | _____ | _____ |
| Revolution counter _____ | _____ | _____ |
| Revolution counter _____ | _____ | _____ |
| Number of revolutions _____ | _____ | _____ |
| Time for above _____ | _____ | _____ |
| Angular velocity of disks_____ | _____ | _____ |
| Angle between holes A and C ( ± ?) | _____ | _____ |
| Distance between disks     ( ± ?) | _____ | _____ |
| Radius of swing of pendulum ( ± ?) | _____ | _____ |
| Calculated velocity of bullet (Eq. 9) | _____ | _____ |
| Observed velocity of bullet (Eq. 10) | _____ | _____ |
| Percent difference | _____ | _____ |
| Kinetic energy of bullet (units?) | _____ | _____ |
| Kinetic energy of (pendulum + bullet) | _____ | _____ |
| Fraction of KE lost (observed) | _____ | _____ |
| Fraction of KE lost (Discussion 9) | _____ | _____ |

Other data:


Approval_____(Is the apparatus in order?)

Hours spent on this experiment: Study_____Lab_____Report_____

EXPERIMENT 51-6
THE ROOF TRUSS

OBJECT

To study the conditions for the equilibrium of a rigid body by determining the external forces acting on a roof truss.

APPARATUS

In Lab: Roof truss consisting of matched parts with one detachable member, one 5-kilogram spring balance, trip scale ($20) and weights, 2-kilogram weights, weight hangers, meter stick, 2-meter stick, table clamp, and cord; pulley fastened to table.

Check out: Vernier caliper, one additional 5-kilogram spring balance, more cord, small weight hanger and weights.

THEORY AND METHOD

A. In order to study the conditions for the equilibrium of a rigid body, in this experiment we investigate the forces acting on a roof truss shown in Figure 1. It is a framework composed of two wooden lever arms hinged at vertex C and resting on the table at points A and B. The end resting at A is held fast by a clamp, but B is mounted on rollers. Cable AB, including the spring balance h, connects endpoints of the lever arms A and B. Any force exerted on one part of the truss by another is called an internal force. Any force exerted on the truss by some outside agency (e.g., the earth, the table, weight hangers, etc.) is called an external force.

B. Briefly, the procedure of this experiment con-



Figure 1. The Roof Truss

sists in (1) hanging known weights on the lever arms, and in (2) measuring other external forces acting on the truss when it is in a motionless state. (A motionless state implies equilibrium, but the converse is not necessarily true.)

C. Any rigid body, such as this truss, must satisfy the three following conditions in order to be in a state of equilibrium:

(a) The algebraic sum of the horizontal components of all the external forces acting upon the structure must be zero, or $\Sigma H = 0$. *

(b) The algebraic sum of the vertical components of all the external forces acting upon the structure must be zero, or $\Sigma V = 0$. *

(c) The algebraic sum of the torques about any axis must be zero, or $\Sigma \tau = 0$. *

That these conditions must be satisfied is clear, since, by Newton's second law of motion, any unbalanced horizontal or vertical forces would result in linear motion of part or all of the system, while any unbalanced torque would result in rotational motion. The truss is, however, in a motionless state, so all three conditions for equilibrium must be satisfied. Since motion is not involved in this experiment, one need not be particular about what units he uses so long as he uses the same units throughout the experiment.

D. It should be emphasized that the same equilibrium conditions apply to separate parts of the truss as well as to the system as a whole, since these parts are themselves rigid and in a state of equilibrium. For instance, if we choose to consider separately one part of the truss, say lever arm BC, we must consider the forces exerted on it by other parts of the system with which it is in contact (e.g., the thrust from arm AC at C) as external forces for arm BC of the truss.

E. Two types of load are considered in this experiment: (I) A symmetrical load with equal weights, $L_1$ and $L_2$ at equal distances $\ell_1$ and $\ell_2$ from A and B, respectively. (II) An asymmetrical load with an additional weight $L'$ at distance $\ell'$ from B or A.

F. Let us consider the vertical and horizontal forces and the torques acting on the truss under both conditions of loading (Figures 2 and 3, page 2).

Vertical forces: The vertical forces acting on the truss as a whole are the weights of the lever arms ($W_1$ and $W_2$), the additional loads or weights ($L_1$, $L_2$, and $L'$), and the upward forces ($A_V$ and $B_V$) of the table supporting the truss. If we consider one arm independently, there will be the additional vertical component $C_V$ of the force exerted upon it by the other arm at C. All except $C_V$ are measurable experimentally. Both $A_V$ and $B_V$ are measured by spring balance v at B, the freely rolling end of the truss. They will be
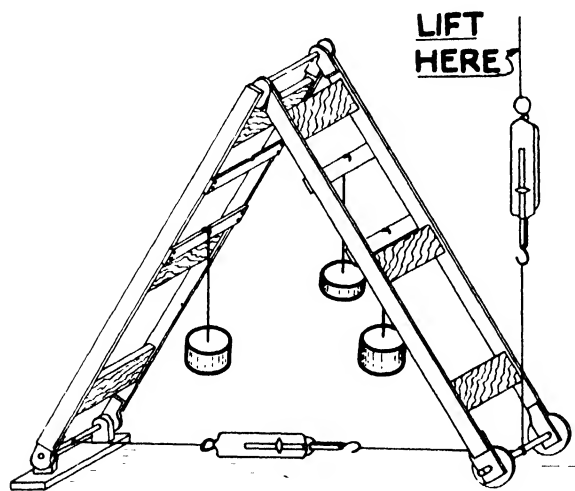
equal for the symmetrical load, and the reading for $B_v$ will represent $A_v$ also. However, when the load is asymmetrical, $A_v$ does not equal $B_v$ and each must be determined separately. Since the truss is itself symmetrical, this may be done in the following manner:

With $L'$ hanging at distance $\ell'$ from B on arm BC, take the reading $B_v$ at B. Then shift $L'$ over to a similar position on AC, a distance $\ell'$ from A. Since the unloaded and loaded arms have merely switched positions, other conditions remaining the same, in the second case the reading at B will represent A for the previous setting $L'$.

Horizontal forces: The horizontal forces acting on the truss, $A_h$ and $B_h$, are measured by spring balance h. If we consider one arm alone, say BC, the horizontal forces acting are the pull of AB at B, i.e., $B_h$, and the horizontal component $C_h$, of the force exerted at C by arm AC.

Attention is called to the fact that the roof truss is constructed so that the axes about which rotation can occur at the corners of the truss are directly in line with the lower edge of the meter sticks forming the truss. Therefore, all distances from the axes may be measured along the lower (reference) edge of the meter sticks.

The weight hangers for loads $L_1$ and $L_2$ hang from brass crossbars. Because of the construction of the crossbars and weight hangers, the point from which the weights are suspended is in the plane of the lower (reference) edges of the meter sticks. Its position can be determined by noting the position of the edge of the brass bar and measuring the depth of notch with the depth gauge of the vernier caliper, used because of its depth gauge rather than for its accuracy.

Torques: All of the forces mentioned may be considered to produce torques about axes represented on a plane diagram by points A, B, or C. The forces themselves are measured experimentally, but in this experiment the torques around any axis are computed as the product of the known magnitude of the force and the distance from its line of action to the axis in question. For example, the torque around B of the weight $W_2$ of arm BC centered at distance $w_2$ from B, is $W_2 w_2 \cos \phi$ where $\phi$ is the angle made by arm BC and the horizontal. Torques also may be taken for the system as a whole or for each arm separately. It can be shown that if $\Sigma H = 0$ and $\Sigma V = 0$, and if $\Sigma \tau = 0$ for any one axis, then $\Sigma \tau = 0$ is true for every axis.

By the use of the three equations $\Sigma H = 0$, $\Sigma V = 0$, $\Sigma \tau = 0$ expressing the conditions of equilibrium, $A_v$, $B_v$, $A_h$, and $B_h$ can be computed theoretically if $L_1$, $L_2$, $L'$, $W_1$, and $W_2$ and their distances from A or B are known. These computed values can be compared with their experimentally determined values to verify the laws of equilibrium.

PROCEDURE

I. Symmetrical Load

1. The roof truss will be found already set up in the lab with the spring balance h connected near the fixed end of the truss. The "outside" of the balance is fastened to the fixed end, and the "inside" to the rolling end. The two beams of the truss already have been "matched" to each other.

2. Hang equal weights $L_1$ and $L_2$ (each about 2.5 kg) on arms AC and BC at equal distances $\ell_1$ and $\ell_2$ from A and from B. Be sure that the weight hangers are also of equal weight. (On some setups the meter sticks read from point C rather than from points A
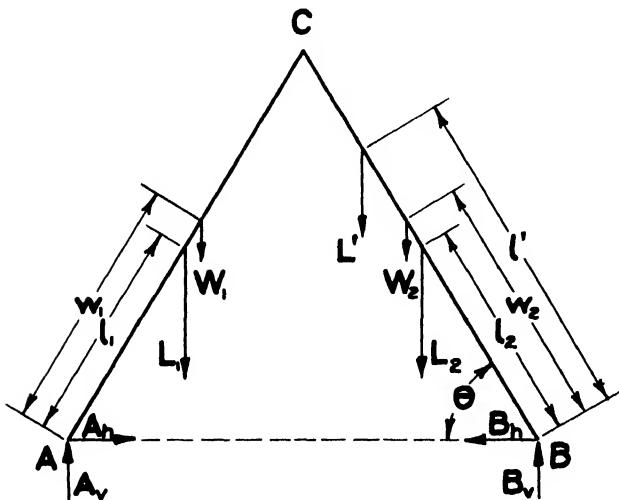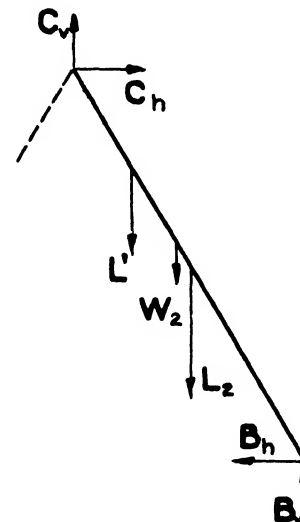


Figure 2



Figure 3

and B; take note of this when you read distances $\ell_1$ and $\ell_2$.) Record values of $L_1$, $L_2$, $\ell_1$, and $\ell_2$, taking due account of the depth of the notches.

3. Adjust the distance AB so that it is greater than 1.0 m and less than 1.4 m.

4. By means of the spring balance v, lift the end B vertically until it is just clear of the roller. Read the spring balance, being sure that the direction of pull is strictly vertical, and read the vertical tension in the balance as $B_v$, and that of the horizontal balance as $B_h$. Evidently $A_v = B_v$, and $A_h = B_h$, since the loads are symmetrical. To be sure the force $B_v$ is vertical, line up a string supporting the spring balance with the corners of the room.

5. With the roof truss resting on its supports again, vary the distance AB until spring balance h reads the value for $A_h$ and $B_h$ found in step 3. Measure the distance AB between the inner corners of the two beams.

6. Repeat steps 2-5 of the Procedure of this experiment with each partner reading the other balance, so that an average of two independent readings of the balances may be obtained. (It is well to jiggle the truss a little so as to overcome any "sticking" due to friction.)

### II. Asymmetrical Load

7. With the roof truss still bearing symmetrical loads, $L_1$ and $L_2$, hang load L (2 kg plus weight of the hanger) on arm BC at distance $\ell'$, greater than $\ell_2$, from B.

8. Adjust the distance AB so that it is greater than 1.0 m and less than 1.4 m.

9. In the same manner as step 4, part I, read spring balance v to get $B_v$. Also, read spring balance h to get $B_h$ and $A_h$.

10. Switch L' to arm AC at the same distance $\ell'$ from A. Read spring balance v at B and record this reading as $A_v$. $A_h$ and $B_h$ should measure the same as they did in step 9.

11. Determine AB as you did in step 5.

### General Instructions for Both I and II

12. Assume that both 5-kg balances are alike. Determine the error in the reading of balance h which was used horizontally. Use either of the following methods: (a) Suspend a weight hanger over the pulley set up in the Lab; hang enough weight on it so that the spring balance reads zero when it is pulling horizontally. (b) Note that the spring balance reads zero when no load is on it irrespective of whether it is right side up or upside down. The conclusion is that both the inside part and the outside part of the balance weigh the same. When the balance is vertical either the inside or the outside stretches the spring; when the balance is horizontal neither stretches the spring. Evidently the correction is equal to the weight of either part of the balance or half the total weight of the balance.

13. Weigh the detachable beam on a trip scale and record the weight as $W_2$. Record the weight $W_1$ given

on the fixed beam. To save you some time in calculating, you are permitted, if you like, to substitute for both $W_1$ and $W_2$ the average of the two.

14. To find the center of gravity of the detachable beam, balance it on the sharp edge of a meter stick and record the distance to B as $w_2$; the position of the center of gravity of the fixed beam has been marked on it so that the distance $w_1$ may be determined easily.

15. Use the average of $\ell_1$ and $\ell_2$, and call it $\ell$. Use also the average of the two readings for $\ell'$ and call it $\ell'$.

16. Leave the truss set up in an orderly condition.

### CALCULATIONS

a. Make all computations indicated on the Data Sheets, and correct the reading of spring balance h, according to step 12 of the Procedure.

b. From the conditions for equilibrium: $\Sigma H = 0$, $\Sigma V = 0$, $\Sigma \tau = 0$, compute the theoretical values for $A_v$, $B_v$, $A_h$, and $B_h$ Indicate each equation algebraically before proceeding to do the numerical work. When you compute torques, be sure to specify the axis about which they act.

### DISCUSSION

1, 2, 3, 4. Discuss such things as the basic ideas illustrated by the experiment, conclusions to be drawn from it, quantitative estimates of sources of error, and possible improvements in the experiment.

5. Discuss the reasons for the position of the spring balance h mentioned in step 1 of the Procedure. What would happen if it were at the other end of the truss, or if the outside of the balance were connected to the rolling end?

6. In step 14 of the Procedure the distance along the edge of the beam of the center of gravity from the pivot was determined, but no attempt was made to find how far away from the reference edge of the meter-stick beams the center of gravity was. Suppose the beam makes an angle of $60°$ with the horizontal, and the center of gravity is 1.0 cm away from the reference edge of the beam. What effect does this have on $w_2$ (or the value that should be used for $w_2$)?

7. Are $A_h$ and $B_h$ always equal to each other?

8. In an experiment similar to part I of this experiment with the symmetrical load, a student read the vertical force $B_v$ as 2 kilograms, but the balance was pulling at an angle of $80°$ with the horizontal, rather than vertically. What errors are caused by this faulty measurement?

9. For a symmetrically loaded roof truss, which meets the table at an angle $\theta$, and has arms of length d, what is the vertical component of the force of AC on BC at C?

10. For the asymmetrically loaded truss with load on the right, show by a sketch the approximate direction of the thrust at C of BC on AC, and show also the direction of the thrust of AC on BC. Label which is which.

Name_____Box No._____Date in Lab_____

Partner_____Lab Section_____Instructor_____

DATA SHEET

Number of Roof Truss_____

Depth of Notches    _____

Length AC_____     Load $L_1$_____     Length $\ell_1$_____

Length BC_____     Load $L_2$_____     Length $\ell_2$_____

Did you take account of the depth of the notches?        _____

Averages:       Load L_____     Length $\ell$ _____

### Symmetrical Load

Measured $B_v$_____     _____ Average_____

Measured $B_h$_____     _____ Average_____

Length AB     _____

### Asymmetrical Load

Length $\ell'$ (first side)_____(second side)_____Average_____

Measured $B_v$     _____     _____Average_____

Measured $B_h$     _____     _____Average_____

Measured $A_h$     _____     _____Average_____

Does $A_h$ read the same in Procedure steps 9 and 10?      _____

Length AB        _____

Horizontal spring balance correction (measured by_____)      _____

Weight $W_2$_____     Weight $W_1$_____     Average W_____

Length $w_2$_____     Length $w_1$_____     Average w_____

Calculated cos $\theta$ for symmetrical load_____

Calculated sin $\theta$ for symmetrical load_____

Calculated cos $\theta$ for asymmetrical load_____

Calculated sin $\theta$ for asymmetrical load_____

SOLUTION FOR THEORETICAL VALUES

Symmetrical Load

For $A_V$ (and $B_V$) use $\Sigma V = 0$. Equation from data:

_____

For $A_h$ (and $B_h$) isolate the beam BC, compute torques about C, and use $\Sigma \tau = 0$. Equation from data:

_____

Simplified equation_____

$B_V$ (measured)_____   (calculated)_____   percent difference_____

$B_h$ (measured)_____   (calculated)_____   percent difference_____

Asymmetrical Load

For $A_V$ compute torques about B on the roof truss as a whole and use $\Sigma \tau = 0$. Equation from data:

_____

Simplified equation _____

For $B_V$ use $\Sigma V = 0$. Equation from data:

_____

For $A_h$ (and $B_h$) isolate the beam BC, compute torques about C, and use $\Sigma \tau = 0$. Equation from data:

_____

Simplified equation_____

$A_V$ (measured)_____   (calculated)_____   percent difference_____

$B_V$ (measured)_____   (calculated)_____   percent difference_____

$A_h$ (measured)_____   (calculated)_____   percent difference_____

Other data:

Approval_____(Is the apparatus in order?)

Hours spent on this experiment: Study_____Lab_____Report_____

EXPERIMENT 51-7
ROTATIONAL ENERGY AND INERTIA

## OBJECT

(I) To verify the principle of conservation of energy for a particular mechanical system whose parts are undergoing changes in potential energy, and linear and rotational kinetic energy.

(II) To show the proportionality of torque and angular acceleration.

## APPARATUS

In Lab: 20-inch diameter painted steel disk mounted in well-lubricated bearings (custodian please note), sparking apparatus set for 5 sparks per second, meter stick, one-inch roll of special waxed paper, trip scale with weights, clock.

Check out: Vernier caliper, ruler, roll of scotch tape, clamp for wax tape, 50-gram and 100-gram weights.

## THEORY AND METHOD

A. Note the steel disk mounted about 9 feet above the floor of the laboratory. Wind a strip of special waxed paper around the outer edge of the disk, securing its end to the rim by a piece of scotch tape--see Figure 1. Hold the disk motionless while you fasten a clamp to the free end of the paper strip and hang weights from the clamp. When the disk is released,
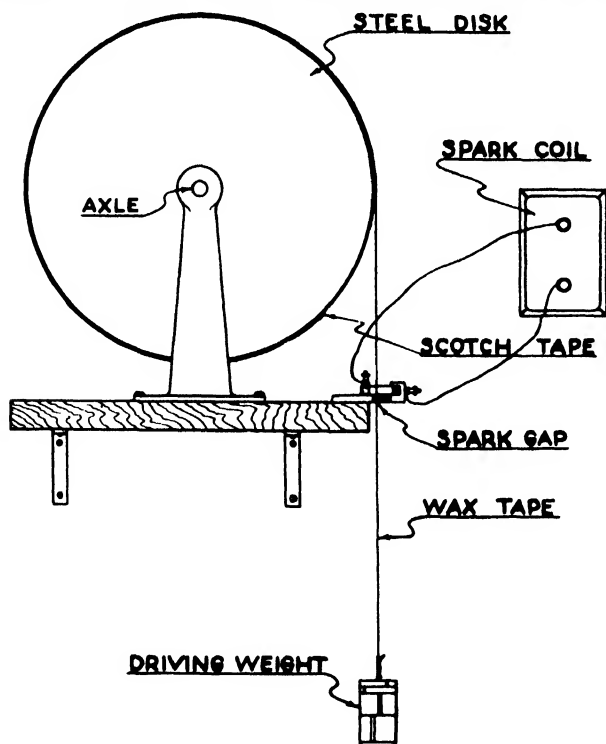


Figure 1. Rotational Energy and Inertia Apparatus

the weights fall, and the disk is made to rotate because of the torque exerted on it by the unwinding paper strip.

B. The first purpose of the experiment is to show that the potential energy lost by the falling weights is equal to the sum of the kinetic energy gained by them and the rotational energy gained by the wheel and axle. This may be expressed by the following relationship:

$$mgh_0 - mgh = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2 \qquad * \quad (1)$$

or

PE lost by weight = KE gained by weight + KE gained by disk     (2)

where $h_0$, $v_0$, and $\omega_0$ represent respectively the height of the weights, from an arbitrary elevation (say, the level of the spark gap), the velocity of the weights, and the angular velocity of the disk, taken at some arbitrary initial time $t_0$; and where h, v, and $\omega$ represent these quantities measured at a later time t. I denotes the moment of inertia of the disk, R its radius, and M its mass. The mass of the weights is represented by m and the acceleration of gravity by g.

All quantities in Equation (1) may be determined experimentally. It is convenient to measure all lengths in meters, masses in kilograms, and times in seconds. Forces, then, are in newtons, and torques in newtons times meters. The value of m for the clamp and weights can be obtained by use of a trip scale, and g may be taken as a known quantity. From the value of M stamped on the disk and a knowledge of R (measured directly or, more accurately, calculated from a measurement of the disk's circumference), I can be computed. Similar measurements can be taken for the additional moment of inertia of the rotating axle. The tape is threaded through a sparking mechanism similar to the one used in Experiment 51-3 on the falling body, and $h - h_0$, v, and $v_0$ may be determined from the spark marks on the tape.

Since

$$v = R\omega \text{ and } v_0 = R\omega_0, \qquad * \quad (3)$$

Equation (1) reduces to

$$mgh_0 - mgh = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + \frac{1}{2}I\frac{v^2}{R^2} - \frac{1}{2}I\frac{v_0^2}{R^2}$$
$$(4)$$

with all terms evaluated experimentally as indicated in the preceding paragraph. Therefore, we can check the right-hand member (side) of Equation (4) against the left-hand member to see if the principle of conservation of energy has been verified.

C. The second purpose of this experiment is to show that the angular acceleration of the disk and axle is proportional to the applied torque, according to the relation:

Torque = (moment of inertia) x (angular acceleration)     * (5)

The torque is varied by using different driving weights, but is not proportional to the driving weight used, as the tension in the tape is not proportional to this weight. Thus for the driving weight of mass m,

$$\text{Weight} - \text{Tension} = ma \qquad *(6)$$
or $\qquad mg - \text{Tension} = ma$
and hence $\qquad \text{Tension} = mg - ma.$

The torque is equal to the tension multiplied by the radius of the disk, thus

$$\text{Torque} = \text{tension x radius} \qquad * (7)$$
or $\qquad \text{Torque} = (mg - ma) R,$

from which it is seen that the torque is not proportional to mg, but rather to mg - ma.

The angular acceleration $\alpha$, is given by

$$\alpha = \frac{a}{r}. \qquad * (8)$$

One either knows or can measure all quantities in Equation (7) for the torque, and in Equation (8) for the angular acceleration so that the values of torque and angular acceleration from these two equations can be substituted into Equation (5) to see if angular acceleration changes in proportion to the torque as the driving weight is changed.

PROCEDURE

1. Students work in groups of three. The instructor will set the spark machine for 5 sparks per second and connect it so that all units are serviced by spark impulses. See that the switch in the primary circuit of your spark coil is open.

. 2. Measure the various dimensions of the disk (or wheel) and its axle (in meters). Note the mass of the disk stamped on it (in kilograms), and also the mass of the axle. Weigh the clamp.

3. With a piece of scotch tape fasten a piece of wax paper to the outer surface of the disk and thread the tape between the electrodes of the spark gap. The tape should be long enough to reach to a point about a foot above the floor. Fasten the clamp and weights on the end of the tape so the total driving weight is about 250 grams weight.

4. Wind up the paper on the disk tightly until the end of the tape is just below the spark gap.

5. The circumference of the disk may be recorded by marking on the tape.

6. Hold the disk motionless until the clamp is also motionless. Keep your hands away from any wires going to the spark coil, close the switch, and release the disk. When the weight approaches the floor, it should tear the tape loose from the disk. Open the

switch. Mark the tape giving the value of the driving weight.

7. Repeat for driving weights of about 500, 400, 300, 200, and 150 grams weight. One student may regard as his the 150 and 300 gram runs, another the 200 and 400 gram runs, and another the 250 and 500 gram runs.

8. On the last run note the time and see how many minutes it takes the disk to stop spinning because of friction. If the time is less than 10 minutes, you must correct for the torque due to friction; see Data and Calculations, part j.

9. All three students should complete parts a to e in the Data and Calculations below before leaving the laboratory, in order to trade their results for these parts. The student with the 150-gram run should choose that run on which to calculate results before leaving the lab.

DATA AND CALCULATIONS

a. From the dimensions of the disk and axle and the values of the masses you may calculate the moment of inertia of the rotating parts using formulae like: moment of inertia of disk or cylinder = 0.5 (mass) (radius)$^2$. You may as well ignore the small holes drilled in the disk to balance it. The larger holes in the disk (about 7.5 inches from the axle) actually do not introduce a serious error because they are located just about at the radius of gyration of the disk. If the holes were filled, the mass and moment of inertia would both increase in almost the same proportion. (The mass of the disk was determined after the holes were drilled. You may wish to verify the accuracy of the mass as stamped on the disk by calculating the mass from the volume of the disk and the known density of steel.)

b. Show that the moment of inertia of the axle is negligible.

c. Measure and record on the Data Sheet the distance of the spark holes in the tape from the original position ignoring holes in the first centimeter (since their positions may not be very accurate). In the case of the slower runs it will be sufficient to measure only every alternate hole (but note that in such cases the time interval has been doubled).

d. Select a run of about 150, 200, or 500 grams weight, and from the distances traveled at successive intervals of time, compute the velocities at the (time) mid-point of the intervals. On engineering cross-section paper plot a graph of velocity as a function of time. Take the instant for which t = 0 as the time when the first spark hole that was one or more centimeters from the rest position was made in the tape. Extrapolate the curve back to the time when the velocity was zero.

e. From the slope of the curve, calculate the linear acceleration of the tape, and hence the angular acceleration of the disk. Record this result, together with the results obtained by your partners, in the places indicated on the Data Sheet.

f. Compute and record the linear acceleration of your second tape, following the method in parts d and e. These data will be used in part h.

g. Select a convenient time near the end of the run used in paragraph d, and from points on the curve determine the quantities h and v in Equation (1). Mark these values on your graph. Select also the values of $h_0$ and $v_0$. Then substitute in Equation (1) and calculate the value of each member. Agreement between the two members may be regarded as verifying the principle of conservation of energy. Should the instructor suggest it, repeat the calculation for your second tape.

h. From your own results for the angular acceleration of the disk, and the two data from your partners, together with the datum that for zero torque the acceleration must be zero, you have five data you can use to plot a graph of torque as a function of angular acceleration of the disk. Presumably you will get a straight line, thereby proving that torque and angular acceleration are proportional.

i. The slope of your graph of part h is equal to the moment of inertia of the disk (since torque $= I\alpha$). Check to see how well it agrees with the calculated value.

j. (Optional at discretion of instructor.) Most of the rotational energy and inertia units have such slight friction that no correction need be made in the experiment for friction. A few of them, however, have frictional errors of from one to two percent. The frictional torque can be assumed to be constant (that is, independent of speed of the disk). This assumption is probably approximately valid so long as the speed is not too great. Students who use the frictional apparatus are required to correct their calculations for friction. Those who use the frictionless equipment are required to correct their calculations for friction also, even though the corrections make no significant change in the results. All students, therefore, have about the same amount of numerical work to do, and also presumably learn about the same amount of physics from the experiment.

Consider first, part g of Data and Calculations concerning Equation (1), which says that the loss of potential energy accounts for two things: the gain in linear kinetic energy of the weights and the gain in rotational kinetic energy of the disk. When corrected for the work done against friction, Equation (1) would be modified by adding to the right-hand member the

work done against friction--(Retarding Force) x $(h - h_0)$. On the Data Sheet there are places to indicate the values of the left and right members of Equation (4), both with and without correction for friction.

Next consider part h of Data and Calculations concerning the proportionality of torque and angular acceleration. Since we have assumed the frictional retarding torque was constant, it follows that the actual accelerating torque in each run was less than supposed by this amount. Replot each value of torque on the same graph, but subtract from it the value of the retarding torque. Does this straighten out the curve appreciably?

## DISCUSSION

1. In this experiment you have done two things: you you verified the law of conservation of energy (Equation 1), and you showed that torque and angular acceleration are proportional. In the second part of the experiment you had five data from which to determine the slope of the line. Actually you could have determined the slope from a single datum in addition to the zero-zero datum. By an algebraic argument (using equations like $v^2 = v_0^2 + 2ah$, $\alpha = \frac{a}{r}$, etc.) show that if torque $= I\alpha$, then necessarily Equation (1) must follow from it. The conclusion is that from a single observation one cannot prove experimentally both Equation (1) and torque $= I\alpha$ (since one follows from the other). Actually in this experiment you verified Equation (1) with one of your tapes and verified torque $= I\alpha$ from the other tapes.

2. What is the mass of a steel disk 0.0127 m thick a and 0.50 m in diameter? What is its moment of inertia about an axis through its center?

3. Suppose that four holes are drilled in the disk of the previous question, each 2.5 cm in diameter and 0.19 m from the axis of the disk. What is the mass of the material removed? What fraction of the original mass is this? Assuming it all to be 0.19 m from the axis, what is the moment of inertia of the material removed? What fraction of the original moment of inertia is this?

4, 5, 6, 7. Discuss the basic ideas illustrated by the experiment, conclusions to be drawn from it, quantitative estimates of sources of error, and possible improvements in the experiment.

Name_____Box No._____Date in Lab_____

Partner_____Lab Section_____Instructor_____

DATA SHEET

Apparatus number or station number                                                  _____

Thickness of disk                 _____  _____  _____   _____

Mass of disk (stamped on it)                                                        _____

Circumference of disk          _____           Radius of disk    _____

Moment of inertia of disk                                                           _____

Length of axle                                                                      _____

Mass of axle                                                                        _____

Diameter of axle              _____           Radius of axle     _____

Moment of inertia of axle                                                           _____

Moment of inertia of disk plus axle                                                 _____

Weight of clamp                                                                     _____

Additional weights added to clamp,    1st run_____         2nd run  _____

Number of sparks per second,          1st run_____         2nd run  _____

Time for disk to stop turning after last run                                        _____

Total driving weight on last run                                                    _____


Continue data on next page. Some spaces for calculations are below.

Time for weight to descend on last run                                              _____

Maximum angular velocity of disk on last run                                        _____

Angular acceleration of disk on last run                                            _____

Angular deceleration of disk on last run                                            _____

Retarding torque (calculated from  torque = $I\alpha$) to produce this deceleration    _____

Retarding force (calculated from  torque = Force x Radius of disk) to produce
    this deceleration                                                               _____

Work done against friction (Data and Calculations, j)                               _____

Equation (1) without friction correction for your first run; value of

    left side_____right side_____percent discrepancy_____

Equation (1) with friction correction (Data and Calculations, j); value of

    left side_____right side_____percent discrepancy_____

DATA SHEET (Continued)

| First Run | Second Run (Optional if instructor so directs) |

Weights added to clamp _____   Weights added to clamp _____

Total driving weight _____   Total driving weight _____

Number of sparks per second _____   Number of sparks per second _____

| Hole No. | Distance from rest position | Velocity at mid-point | Hole No. | Distance from rest position | Velocity at mid-point |
|---|---|---|---|---|---|
| ___ | _____ | _____ | ___ | _____ | _____ |
| ___ | _____ | _____ | ___ | _____ | _____ |
| ___ | _____ | _____ | ___ | _____ | _____ |
| ___ | _____ | _____ | ___ | _____ | _____ |
| ___ | _____ | _____ | ___ | _____ | _____ |
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| ___ | _____ | _____ | ___ | _____ | _____ |
| ___ | _____ | _____ | ___ | _____ | _____ |
| ___ | _____ | _____ | ___ | _____ | _____ |
| ___ | _____ | _____ | ___ | _____ | _____ |

Linear acceleration of tape (I)_____ Linear acceleration of tape (II)_____

Linear acceleration of partner's tape (III)_____total driving weight_____

Linear acceleration of partner's tape (IV)_____total driving weight_____

Angular acceleration of disk (I)_____(II)_____(III)_____(IV)_____

Approval_____

Hours spent on this experiment: Study_____Lab_____ Report_____

EXPERIMENT 51-8

SIMPLE PERIODIC MOTION

## OBJECT

To study simple periodic motion.

## APPARATUS

In Lab: Hooke's law apparatus consisting of vertical support with mirror and scale; special simple periodic motion apparatus with spring, mass, 30-sparks-per-second sparking device, slider, and special wax-paper spark tape of 5 inches width.

Check out: "White" spring, piece of string, weight holder, four 50-gram weights or equivalent, clock, ruler.

## THEORY AND METHOD

A. Hooke's law states that if a deforming force is applied to a body and if its elastic limit is not exceeded, the deformation is proportional to the deforming force. As an example, the stretch or elongation $x$ of a spring is proportional to the stretching force $G$, that is,

$$G = kx \qquad \qquad * (1)$$

where $k$ is a constant and is called the force-constant of the spring. This law may be verified by applying successively increasing forces to a spring and measuring the resulting increases in length. A graph of the stretching force $G$ as a function of the elongation $x$ is a straight line, the slope of which is equal to the force-constant $k$. The force-constant may be determined also by measuring the period of vibration of a mass hanging on the spring, and by making use of some theoretical arguments given below. Agreement between the two values of force-constant so obtained for the same spring may be taken to validate the theory.

B. Figure 1 shows a particle $P$ moving counter-clockwise in a circle of radius $r$ with a constant angular velocity $\omega$. Consider the horizontal projection of this particle on the vertical diameter. As $P$ moves around the circle, its projection or shadow $S$ moves vertically up and down with what is known as simple periodic motion (sometimes called simple harmonic motion). At any time the displacement $y$ of the shadow $S$ from the mid-point is

$$y = +r \sin \vartheta \qquad \qquad * (2)$$

where $\vartheta$ is the angle made by the radius vector OP from the center of the circle to the particle $P$ with the reference line OA. The sign is taken as positive because sin $\vartheta$ and $y$ have the same sign in all quadrants; for instance, when $\vartheta$ is in the first quadrant, sin $\vartheta$ is positive and $y$ is upwards or positive.

C. Since the velocity $V$ of the particle is $r\omega$ directed along the tangent to the circle, the velocity $v$ of the shadow is the vertical component of $V$, or

$$v = +r\omega \cos \vartheta \qquad \qquad * (3)$$

The sign is taken as positive because the velocity and cos $\vartheta$ have the same sign in all quadrants; for instance, the velocity is upward (positive) when $\vartheta$ is in the first quadrant and hence cos $\vartheta$ is positive.

D. Since the acceleration $A$ of the particle $P$ is $r\omega^2$ --or $v^2/r$ --directed toward the center, the acceleration $a$ of the "shadow" is the vertical component of $A$, or

$$a = -r\omega^2 \sin \vartheta. \qquad \qquad * (4)$$

The sign is taken as negative because the acceleration and sin $\vartheta$ have opposite signs in all quadrants; for instance, the acceleration is downward (negative) when $\vartheta$ is in the first quadrant and hence sin $\vartheta$ is positive.

E. If the angle $\vartheta$ increases uniformly with the time $t$ then $\vartheta = \omega t$, where $\omega$ is the angular velocity of particle $P$, and Equations (2), (3), and (4) may be written

$$y = r \sin\omega t, \qquad \qquad * (5)$$
$$v = r\omega\cos\omega t, \qquad \qquad * (6)$$
$$a = -r\omega^2\sin\omega t. \qquad \qquad * (7)$$

[ Students who have had differential calculus will recognize that Equation (6) is the derivative of Equation (5) and Equation (7) is the derivative of Equation (6). Thus the velocity is the rate of change of displace-
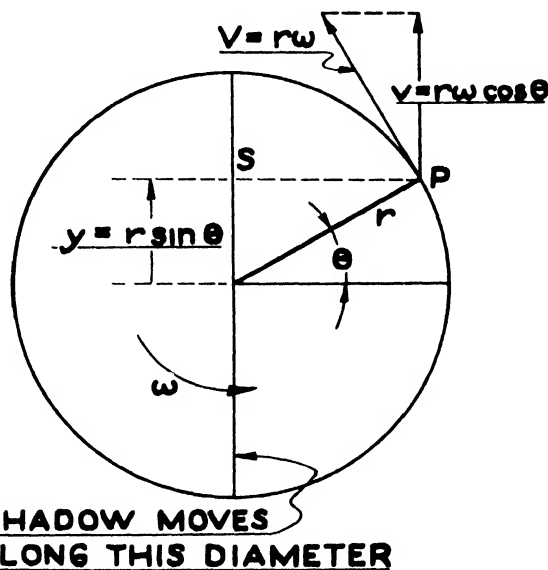


Figure 1. The shadow $S$ of the particle $P$ moves vertically with simple periodic motion.

ment with time and is $\frac{dy}{dt}$. The acceleration is the rate of change of velocity with time, that is, $\frac{dv}{dt}$ or $\frac{d^2y}{dt^2}$ . Incidentally, it is much easier to obtain Equations (6) and (7) from Equation (5) by differentiating in one or two steps, than it is to remember the geometrical argument for getting them. This is an example of the greater power of calculus over algebra and geometry. ]

F. It is obvious from Equations (2) and (4) that

$$a = -\omega^2 y. \qquad \qquad * \quad (8)$$

Since $\omega^2$ is necessarily a positive quantity, it follows that the acceleration of the "shadow" is proportional to its displacement from the mid-point, and is always directed to the mid-point, i.e., the acceleration is down if the displacement is up, and vice versa. Simple periodic motion is defined as being that motion for which the acceleration is negatively proportional to the displacement from the mid-position.

G. Now consider a mass $m$ hanging on the end of a vertical spring having a force-constant $k = G/y$, where $G$ is the force that must be exerted on the spring to stretch it from its equilibrium length a distance $y$. It is evident that the force $F$ exerted by the spring on the mass is in the opposite direction, so we may write

$$F = -ky \qquad \qquad * \quad (9)$$

Most students recall the relation among the force exerted on an object, its mass, and its acceleration, $F = ma$. Substituting $ma$ for $F$, we have $ma = -ky$ or

$$a = - \frac{k}{m} y \qquad \qquad (10)$$

Since $k$ and $m$ are constants for any given experiment, it follows that the acceleration of the mass is proportional to its displacement from the mid-point, and is always directed toward the mid-point, i.e., the acceleration is down if the displacement is up, and vice versa.

H. This condition mentioned in the previous paragraph is exactly the same as for the "shadow." We thus have two identical kinds of motion in which the acceleration is proportional to the displacement, but oppositely directed. The acceleration of one is given by Equation (8), the other by Equation (10). Since they are the same, they must be equal, and we have

$$-\omega^2 y = - \frac{k}{m}y \quad \text{or} \quad k = m\omega^2.$$

But $\omega = 2\frac{\pi}{T}$ where $T$ is the period of vibration for the "shadow," and hence of the mass on the spring (since their motions are identical). Therefore,

$$k = 4\frac{\pi^2 m}{T^2} . \qquad \qquad (11)$$

By measuring the period $T$ of the vibrating mass $m$, one may calculate $k$ and compare it with the value obtained by the stretching procedure discussed in section A. If one plots a graph of $T^2$ against $m$, he should get a straight line, since

$$T^2 = \frac{4\pi^2}{k} m, \qquad \qquad (12$$

and the slope should be $\frac{4\pi^2}{k}$ from which $k$ can be found.

I. We have made the tacit assumption in the preceding argument that the mass of the spring could be overlooked. Actually, of course, it has some mass which influences the motion somewhat. By an argume too long to give here, it can be shown that about one-third the mass of the spring should be added to the mass $m$, if Equation (12) is to be obeyed exactly. (I the spring has a mass small compared to the mass c the stretching weights, as is the case for most of the experiments, then exactly one-third of the mass of t spring should be added.) You will find, when you plot $T^2$ against $m$, that the straight line does not go through the origin. In fact, the line can be extrapolat so that when $T = 0$, $m$ is negative by an amount corresponding to the correction factor mentioned in this paragraph. The slope of the line is given correc by Equation (12), even though the line is displaced fr the origin.
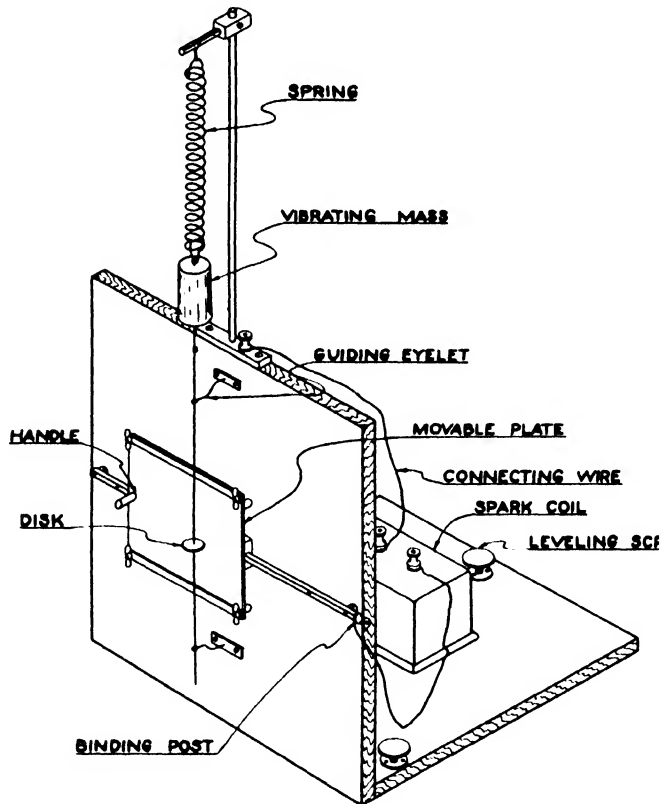


Figure 2. Simple Periodic Motion Apparatus

Caution: When you touch the handle of the plate do n get your fingers near the "hot" disk.

J. If agreement is obtained for the value of the force-constant as determined by the two different methods, namely, the stretching method and the vibrating method, then the theory may be considered to be verified--including such fundamental matters as F = ma on which the theory depends.

K. We shall use the apparatus shown in Figure 2 to verify the general properties of simple periodic motion described above. As the mass vibrates up and down on the end of the spring, the mass and the disk suspended from it will describe simple periodic motion. The movable aluminum plate is covered with wax paper, and a sparking device passes a spark from the disk through the paper to the grounded aluminum plate every one-thirtieth of a second, recording the vertical position of the disk. The plate and paper are moved slowly past the disk, in order to make clear the sequence in which the holes are made in the paper. From the recorded dots we shall plot curves for y, v, and a, as functions of t. We shall compare these with curves predicted by Equations (5), (6), and (7).

PROCEDURE

Note: About two pairs of students should begin with parts 1-6; the rest of the class should start with part 7 and interrupt their work on parts 7-13 of the experiment to do parts 1-6 whenever the apparatus is not in use.

1. The sparking device shown in Figure 2 will be connected by the instructor to give 30 sparks per second between the disk on the rod fastened below the mass hanging on the spring and a movable (grounded) aluminum plate to which may be clamped some special wax paper used for recording positions of the sparks. The instructor will see that the spring hangs vertically with the rod hanging freely in the guiding-hook eyes. He will clamp the apparatus in position. Do not disturb the adjustments. Clamp a short length of paper vertically and tightly over the movable aluminum plate. Three or four papers may be placed under the clamps at one time, thus giving several records, one for each student.

2. Turn on the spark-motor switches if they are not already on.

3. Set the mass vibrating vertically with approximately the maximum possible amplitude.

4. (Caution.) Close the switch in the primary circuit of the spark coil, but do not touch any of the wiring or the spring or the vibrating mass while the spark switch is closed.

5. Move the aluminum plate to which the paper is fastened at a moderate speed so as to draw the entire width of the wax paper across in front of the sparks. The plate should be moved so as to get a record of about one and one-half or two complete vibrations. Immediately open the spark switch. If you moved the plate too fast, or too slowly, try again with fresh paper. Perhaps you will practice moving the plate at the proper speed before wasting too much paper.

6. Stop the mass from vibrating and locate the equilibrium position of the disk. You can mark it on the paper most easily by moving the paper and plate past the disk while the mass is at rest and while the sparks are turned on. Open the spark switch; turn off the motor; and remove the paper from the aluminum plate. This concludes the work with this part of the equipment.

7. Determine the mass of the white spring on the trip scale. (See note.)

NOTE: If you have a red spring instead of a white one, modify the numbers in the later steps of the experiment as follows: In step 8 change 100 grams to 200 grams; in steps 11 and 12 change 50, 100, 150, 200, and 250 to 100, 200, 300, 400, and 500 grams.

8. Hang the white spring on the hook at the top of the Hooke's law apparatus, and hang 100 grams (including the hanger) on the bottom of the spring. You may need to adjust the leveling screws so the apparatus is vertical.

9. Set the mass vibrating vertically with an amplitude of from 2 to 5 cm and attempt to time 50 vibrations. Probably you will find that the spring begins to swing as a pendulum with a period simply related to that of the vertical vibrations, and that consequently it is impossible to time 50 vertical vibrations.

10. Insert a 40-cm length of string between the spring and hook. Repeat step 9 for an amplitude of about 1 cm. Then repeat, but with an amplitude of about 4 cm.

11. Repeat step 10 for masses of 50, 100, 150, 200, and 250 grams with any convenient amplitude.

12. Sighting over some convenient point, such as the top of the hanger, measure the position of the point on the scale for spring loadings of 0, 50, 100, 150, 200, and 250 grams. To eliminate parallax, line up the point with its reflection in the mirror. After completing the series of readings, check the first reading to be sure the elastic limit of the spring has not been exceeded.

13. Repeat as many parts of steps 11 and 12 as you feel are necessary in order to give you the accuracy you think necessary.

NOTE: The experimental part of the work in this experiment ordinarily can be completed by freshmen in about an hour or less. If students will go about the business of plotting the curves with some vigor, they will find they can do all the apparently forbidding quantity of arithmetic in another hour. If, however, they spend an hour worrying about how to think up a good excuse for not doing it, they will take a longer time. The instructor probably will be willing to accept work based on alternate spark holes, rather than on all spark holes.

DATA AND CALCULATIONS

a. Make small circles around each hole in your wax paper of steps 1-6 and draw lines through each hole across the tape perpendicular to the edge of the paper.

b. Pick out some hole near the edge of the paper and call it point 1. Measure its displacement from the

equilibrium position and record it on the Data Sheet. Distances above the equilibrium position are counted positive; those below, negative.

c. Measure the displacement of the next hole (point 2) from the equilibrium position and record it. Continue in this way until at least two holes more than a complete vibration have been recorded.

d. With dots or small circles to indicate points, on engineering cross-section paper having 10 lines per cm, plot a graph of the successive positions of the vibrating mass as a function of time, using a scale of 5 mm per one-thirtieth of a second. You should get a sine curve. Because of errors in your measurements and erratic behavior of the spark your points on the graph may be a bit irregular. Draw a smooth curve, and then record on the Data Sheet the values of the smoothed-out distances of your holes from the equilibrium position. The units are cm displacement plotted as a function of time in one-thirtieths per second intervals.

e. The difference in distance between successive points gives the average velocity during the time interval. Calculate these so-called first differences (of the smoothed displacements) and enter them in the next column--that is, subtract each displacement from the one above it and enter the result in the next column with the proper sign. You must assume that the average velocity values are the velocities at the mid-point of the time interval, which is not quite true, inasmuch as the acceleration is not constant. Plot these velocities as a function of time on the same sheet of graph paper as you used for the displacements. Remember that the velocity points (which should be indicated by small plus signs [ + ] to distinguish them from the displacements) should be plotted midway in time between the displacement points. The two curves may be distinguished also by the use of colored pencils. Choose the ordinate scale so that the curve fills most of the paper. Your units for the velocities or ordinates of this second curve are cm per one-thirtieth of a second. If you wish to convert to cm per sec, multiply by 30.

f. If the displacement curve is a true sine curve (as it should be, except for a very small amount of damping, due to friction), then the curve representing first differences is a true cosine curve, but its amplitude does not have exactly the proper value, unless the points on the sine curve are spaced infinitesimally. In this experiment the points are sufficiently close together that any error introduced by finite spacing of points is very small.

g. Draw a smooth curve through the velocity points and then enter in the next column the values of the smoothed velocities.

h. Calculate the second differences of the displacement--or to be more precise, the first differences of the smoothed velocity column. The second differences of the displacement, or the first differences of the velocity, represent the acceleration (subject to the limitation mentioned in part f). Again, on the same graph,

plot the second differences using crosses (x) to indicate points. Remember to plot the acceleration points midway in time between the velocity points. Choose a scale of ordinates so that the curve covers most of the paper. Your units for this curve are cm per $\frac{1}{30}$ sec per $\frac{1}{30}$ sec. If you wish to convert to $\frac{cm}{sec^2}$, multiply by 900.

i. The acceleration points are likely to be rather irregular (since small errors in the displacement curve are magnified in the velocity curve, and magnified again in the acceleration curve). By using some care probably you can draw a fairly smooth curve for the acceleration, even if the points are scattered rather badly. Take some care to locate those points where the acceleration curve has its maximum and minimum positions, as well as where it crosses the axis.

j. Determine the amplitudes of your displacement, velocity, and acceleration curves. Determine, also, the period of the vibrations by counting the number of $\frac{1}{30}$ -second intervals in a complete vibration. How well do the amplitudes of the velocity and acceleration curves compare with the amplitudes expected from Equations (5), (6), and (7)? If your curves for velocity and acceleration (allowing for errors in plotting) seem to be a reasonable approximation to cosine and sine curves, respectively, and if they have approximately the proper amplitude, then you may conclude that the theory leading to Equations (4), (5), and (6) has been verified.

k. NOTE: By plotting these curves you have shown that if the displacement follows a sine law, the velocity follows a cosine law, and the acceleration follows a sine law of opposite sign to the displacement. (If you have any doubt as to what a sine curve or cosine curve should look like, you might plot one on a separate piece of paper for comparison purposes.) It is easily seen from your graphs that when the velocity is a maximum the acceleration and displacement are zero, and vice versa, etc. Study your curves until you understand what they really mean. Probably without knowing it, you have done what is called differentiating curves graphically. If you study a course in numerical methods in mathematics, you will learn more about his procedure.

l. Determine the period T of the vibration for each of the masses used in step 11, and plot a curve of $T^2$ as a function of the mass. Determine the slope of the line and give some estimate of the precision of your value for k calculated from Equation (11).

m. Extrapolate your graph of $T^2$ against m and determine the m intercept. Compare this value with one-third the mass of the spring.

n. Plot a curve of the stretching forces of step 12 as a function of the increase in length of the spring from its unloaded state. Determine the slope of the line, and then from Equation (1), G = kx, calculate k. Give some estimate of the precision of your value for k.

o. Compare the values of k obtained in parts $\ell$ and n.

DISCUSSION

1,2,3,4. Discuss such things as the basic ideas illustrated by the experiment, the conclusions to be drawn from it, quantitative estimates of sources of error, and possible improvements in the experiment. Note: This experiment illustrates more than the usual number of physical principles.

5. If you have definite opinions about it, comment on the relative accuracy in the two methods for determining k.

6. When the velocity of a particle undergoing simple periodic motion is negative and is increasing in a negative sense, is its acceleration positive or negative and is it increasing or decreasing? Explain in some detail.

7. When the displacement of a particle undergoing simple periodic motion is positive and is decreasing, is its acceleration positive or negative and is it increasing or decreasing?

8. When the displacement is a maximum, what is the velocity?

9. A particular airplane instrument panel vibrates approximately in simple periodic motion with an amplitude of $\frac{1}{64}$ inch at the fundamental frequency of the motor 2400 $\frac{rev}{min.}$. What is the maximum acceleration of the instruments? Compare this acceleration with the acceleration of gravity.

10. Explain which of the following motions are approximately simple periodic motion and which are not: the motion of the earth around the sun, the swing of a pendulum bob, the vibration of a violin string.

Name_____Box No._____Date in Lab_____

Partner_____Lab Section_____Instructor_____

EXPERIMENT 51-8, SIMPLE PERIODIC MOTION                                    6

DATA SHEET

| Point | Displace-ments (cm) | "Smoothed" Displace-ments (cm) | First Differences (velocity cm per 1/30 sec) | Smoothed Velocities | Second Differences (acceleration cm per 1/900 sec²) |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | | | | | |
| 8 | | | | | |
| 9 | | | | | |
| 10 | | | | | |
| 11 | | | | | |
| 12 | | | | | |
| 13 | | | | | |
| 14 | | | | | |
| 15 | | | | | |
| 16 | | | | | |
| 17 | | | | | |
| 18 | | | | | |
| 19 | | | | | |
| 20 | | | | | |
| 21 | | | | | |
| 22 | | | | | |
| 23 | | | | | |
| 24 | | | | | |
| 25 | | | | | |

## DATA SHEET (Continued)

| Point | Displacements (cm) | "Smoothed" Displacements (cm) | First Differences (velocity cm per 1/30 sec) | Smoothed Velocities | Second Differences (acceleration cm per 1/900 sec²) |
|-------|------|------|------|------|------|
| 26 | | | | | |
| 27 | | | | | |
| 28 | | | | | |
| 29 | | | | | |
| 30 | | | | | |
| 31 | | | | | |
| 32 | | | | | |
| 33 | | | | | |
| 34 | | | | | |
| 35 | | | | | |

Period of curves for displacement_____velocity_____acceleration_____

Average value of T _____ Value of $\omega$ _____

Amplitude of displacement curve  measured_____calculated_____percent difference_____

Amplitude of velocity curve  measured_____calculated_____percent difference_____

Amplitude of acceleration curve  measured_____calculated_____percent difference_____


Mass of "white" spring_____

What happens in step 9?_____

Step 10, time of 50 periods    1 cm ampl._____ 4 cm ampl._____

| Mass on end | 50 Periods | | Period Sec. | (Period)² | Scale Reading |
|-------|------|------|------|------|------|
| 0 g | | | | | |
| 50 | | | | | |
| 100 | | | | | |
| 150 | | | | | |
| 200 | | | | | |
| 250 | | | | | |

Value of k part $\ell$ _____ part n _____percent difference_____


Approval_____

Hours spent on this experiment: Study_____Lab_____Report_____

## OBJECT

To discover by reason and by experiment the formula for the period of a torsion pendulum.

Note: Most experiments in freshman physics courses have one of the following kinds of objects: (1) to determine some physical quantity, such as the speed of the rifle bullet; or (2) to verify some theory or formula, such as the formula for centripetal force; or (3) to give the student practice in using some instrument, such as a micrometer. This experiment is unique in that it has as its object to discover something by a scientific investigation. It is suggested that students bring to the laboratory their slide rule, about 4 sheets of engineering cross-section graph paper, and 2 sheets of 2 x 2 cycle log-log graph paper, but that before coming to lab they read this experiment only as far as the end of the section on Theory and Method.

## APPARATUS

In Lab: Clamp for torsion pendulum, trip scale, meter stick, demonstration torsion pendulum set-up.

Check out: Torsion-pendulum plate, two rings (diameters approximately 10 and 15 cm), two steel wires about 65 cm long (diameters about 0.05 cm and 0.08 cm), one brass wire, clock, slotted weights, weight holder, light string about 30 cm long, Vernier caliper, micrometer caliper.

## THEORY AND METHOD

A. Figure 1 is a diagram showing a torsion pendulum similar to the one used in this experiment. It consists of a bob or plate suspended from a wire (or rod) so that any rotational displacement of the bob around the axis of the wire will produce a proportionate restoring torque in the wire in accordance with Hooke's law. If the bob is twisted from its equilibrium or rest position and then released, it will describe an angular harmonic motion around the wire as axis, with its original rest position as mid-point.

B. The torsion-pendulum wire may be either steel or brass. The plate may be loaded with either one or both of two rings which have different moments of inertia. They fit in the grooves on the plate shown in Figure 1. The length of the supporting wire below the clamp may be varied at will. The period of the torsion pendulum is determined by comparison with a clock having a sweep-second hand.

C. Since it is often impossible to infer formulas and laws directly from experimental data, various systematic means for interpreting experimental data have been developed to aid both the student in his simple experiments and the seasoned experimenter in his more complicated work. In this experiment we make use of two such methods: dimen-
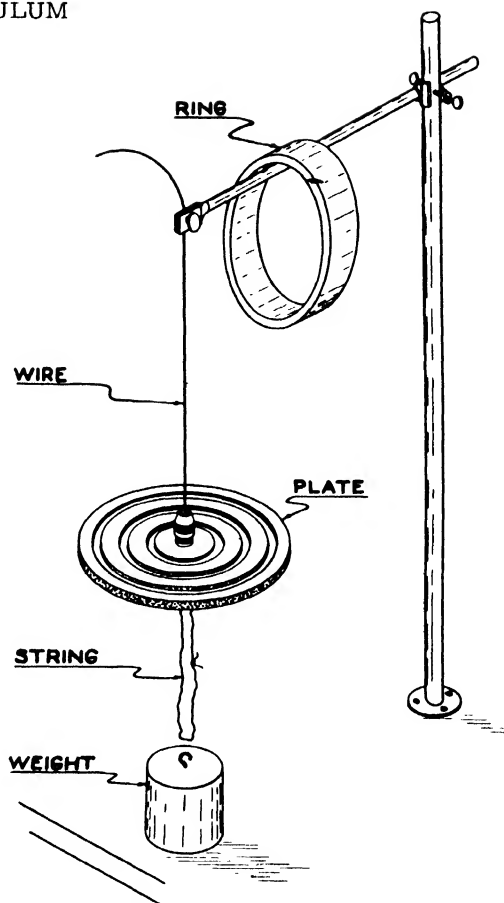


Figure 1. The Torsion Pendulum

sional reasoning and graphs. They are discussed in the next two sections.

D. Dimensional Reasoning. Suppose you have forgotten the formula for centripetal force F, and no reference book is at hand, but you remember the formula involves some combination of the mass m, the speed v, and the radius r, not preceded by any numerical constants. It is evident that the dimensions of the quantities on both sides of the equation must be equal. Letting the symbol (L) stand for the unit of length, (M) for the unit of mass, and (T) for the unit of time, and recalling that Force = mass x acceleration, the dimensions of force are clearly $\frac{(M)(L)}{(T)^2}$, or $(M)(L)(T)^{-2}$. The combination of m, v, and r must have the same dimensions.

Assume that $F = m^x v^y r^z$, where the unknown exponents are indicated by x, y, and z. Recalling that velocity has the dimensions $\frac{(L)}{(T)}$ or $(L)(T)^{-1}$, we may substitute the dimensions of $m^x v^y r^z$ and get

$$(M)^x (L)^y (T)^{-y} (L)^z$$

which must be the same as the dimensions for F; that is,

$$(M)(L)(T)^{-2}.$$

Comparing exponents: From the exponents for (M) we have x = 1, from the exponents for (L), y + z = 1, and for the exponents for (T), -y = -2. These three simultaneous equations in x, y, and z can be solved at a glance and yield the result.

$$x = 1, \quad y = 2, \quad z = -1.$$

Therefore, the required formula for centripetal force must be F = $m^1 v^2 r^{-1}$, which is usually written

$$F = \frac{mv^2}{r}.$$

There is nothing in the dimensional argument to say whether or not there may be a numerical constant in front of the symbols (such as 2 or π or something else). Though dimensional reasoning can give only the exponents of the factors and does not tell about constants, it is often a very useful kind of analysis. In this experiment some use is made of dimensional reasoning.

E. <u>Graphs</u>. Graphs are often used in experimental work to determine the nature of the dependency of one variable quantity upon another. Consider again the example of centripetal force. Keeping m and r constant, to find how F varies with v we plot a graph of F as a function of v, the points on the graph representing experimental data. We note that the curve passes through the origin (i.e., zero force for zero speed) and is concave upwards. A person acquainted with analytic geometry may guess immediately the possibility of a parabolic relation between F and v, but even if one is not good at guessing, there is a way to find out just what the relation is. Plot curves of

F against $\frac{1}{v}$, F against $v^2$, $F^2$ against v, F against $\frac{1}{v^2}$, etc., until in some graph you get a straight line, in this case for F against $v^2$. You then have the proper relation between F and v; namely, that F is proportional to $v^2$. In studying for this experiment you will find it helpful to review the section on Graphs. In that section additional ideas concerning graphs are discussed.

F. As further preparation for this experiment, review the definitions of Young's modulus and the shear modulus.

G. A list of all the possible things that might appear in the formula for the period of a torsion pendulum would include things such as this:

a. The length of the torsion wire
b. The radius (or diameter) of the torsion wire
c. The density of the torsion wire
d. The shear modulus of the torsion wire
e. Young's modulus for the torsion wire
f. The mass of the plate
g. The density of the plate
h. The thickness of the plate
i. The weight of the plate
j. The moment of inertia of the plate
k. The moment of inertia of the rings on the plate
l. The weight hanging on the torsion wire
m. The acceleration of gravity
n. Atmospheric pressure
o. Room temperature
p. The humidity
q. The intensity of illumination in the room
r. The color of paint on the apparatus
s. The amplitude of vibration
t. Numerical constants

H. Probably some of these 20 items seem farfetched, and some of them are. In this experiment we shall find out which ones are not relevant both by reason and by experiment. We shall also find out which ones are important, and just how they enter the formula. Finally, we shall do our best to determine the value of the numerical constant appearing in the equation.

I. It should be remarked that although the formula for the torsion pendulum can be made to sound plausible without the use of calculus, calculus is required for a rigorous derivation. Since most freshmen have not had calculus, the approach in this experiment is not that of verifying a formula derived in a mysterious way by calculus, but rather of trying to discover just what it is. The student may, of course, hunt up the formula in a reference book (it is not given explicitly in the text), but in that case he will be prevented from deriving the most benefit from the examples of the experimental approach and the scientific method used in this experiment.

J. The Procedure involves six experiments (such as, varying the length of the torsion wire) and various calculations and graphs associated with these experiments.

The lab quiz will not include material beyond this sentence; it is suggested that students <u>not</u> read beyond here before they come to the lab.

PROCEDURE

1. General Measurements. Determine the mass (in kg) of the two rings of the torsion pendulum on the trip scale (and record the data on the Data Sheet). Measure the inner and outer diameters of the rings (in meters). Measure the diameter of the plate, and determine its mass on a trip scale. With the micrometer, measure very carefully the diameter of each of the three wires in about five different places. Take special care to get accurate values for the diameter. Record also the radius, as it is used more customarily in the torsion pendulum formula than the diameter. From the dimensions and masses of the rings, calculate their moment of inertia I from the following (exact) formula: $I = \dfrac{M(r_i^2 + r_o^2)}{2}$ where M is the mass of the ring, and $r_i$ and $r_o$ represent the inner and outer radii of the ring Assuming the plate to be a uniform disk, make a rough estimate of its moment of inertia ($I = \frac{1}{2}MR^2$).

2. Set-up. Using the steel wire having the larger diameter, set up the torsion pendulum, examining the demonstration torsion pendulum, or asking the instructor for assistance if necessary. Clamp the torsion wire so that it is 0.400 m long. Take care not to get kinks or bends in the wire. Place the ring with the larger diameter on the plate.

3. Experiment I: Effect of amplitude of vibration (item "s" of section G). Twist the plate through an angle of about $45°$ and release it. Determine the period by timing (in seconds) 20 vibrations (or 10 vibrations if you are careful). Repeat for amplitudes of $90°$ and $180°$. Record the data. If these determinations are made with care, you can use the data for calculations later without having to repeat the experiment for better accuracy. Estimate to half a second or better.

Interpretation: What is your conclusion as to the effect of amplitude of vibration upon the period? (Answer on the Data Sheet.)

4. Experiment II: Effect of length of the torsion wire (item "a"). Using any convenient amplitude, say $90°$, time 20 (or 10) vibrations for the following lengths: 0.60 meter, 0.30 m, 0.15 m. Record the data.

Interpretation: Using about a quarter of a sheet of ordinary graph paper, plot the period P as a function of the length $\ell$. If the resulting curve is not smooth, you may need to repeat some of the parts of this experiment or Experiment I. The curve should pass through the origin. (Why?) Try plotting P or $P^2$ against various powers of $\ell$, (see section E in the Theory and Method) until you get a straight line through the origin. What do you conclude about the relation between P and $\ell$? Perhaps you will prefer to use your log-log paper to determine the exponent.

5. Experiment III: Effect of the weight hanging on the torsion wire (item "l"). To determine if the angular motion of the weight is of importance, or if mere weight has an influence, attach a string to the hook on the bottom of the plate. Hang weights from the lower end of the string approximately equaling the weights of the ring with the smaller diameter that is not now on the plate. If the string is about 0.3 m long, it will absorb the twist of the plate, so that the weights on the lower end will remain sensibly at rest. In a following experiment the ring will be substituted for the weights, so that the weight on the torsion wire will be about the same, but in that case the ring will rotate with the plate. Time 20 (or 10) vibrations, and check the result against the appropriate datum from Experiment II. Record the data.

Interpretation: What do you conclude about the relation between P and the weight hanging on the wire?

6. Experiment IV: Effect of moment of inertia of the plate and of the rings on the plate (items "j" and "k"). It is easy to reason that the period of the torsion pendulum is affected by the moment of inertia of the plate plus rings, but that the torsion wire reacts the same whether this moment of inertia be due to the plate or to the rings. Reset the length of the wire to 0.400 m. The period already has been determined when the larger ring is on the plate (Experiment I). There are three other possibilities, each of which should be timed once; they are, the plate alone, the plate plus the smaller ring, and the plate plus both rings. Record the data. These determinations should be made particularly carefully as they are the basis of a graph from which ultimately the value of the numerical constant in the formula (item "t") will be calculated. For this reason, if you still have as much as an hour in the laboratory period remaining, it will be well to repeat these measurements in order to get good averages.

7. Graphical Interpretation of Experiment IV: Call the moment of inertia of the plate $I_0$, that of the smaller ring $I_1$, and that of the larger ring $I_2$. The periods have been determined for four different values of moments of inertia: $I_0$, $(I_0 + I_1)$, $(I_0 + I_2)$, and $(I_0 + I_1 + I_2)$. As you do not know exactly the value of $I_0$, plot on about a quarter of a sheet of graph paper, on the right-hand side of the paper, the period of vibration as a function of the moment of inertia minus $I_0$. You have then four points: $(I_0 - I_0)$ or zero, $I_1$, $I_2$, and $I_1 + I_2$. You should get some sort of smooth curve. Leave room enough to extrapolate the curve well back to the left. The curve will appear to be nearly straight, but if you have done your work carefully it will be seen to be concave downward.

Try plotting P or $P^2$ against various powers of $(I - I_0)$ until you get a straight line, which should cut the horizontal axis to the left of the origin near $-I_{0 \text{ calc}}$ where $I_{0 \text{ calc}}$ represents the approximate value of the moment of inertia of the plate calculated

at the outset of the experiment. Replot these data on a whole sheet of graph paper, leaving room to extrapolate the line back to the horizontal axis. What do you conclude about the relation of P to moment of inertia of plate plus rings? Record your value for the intercept of the curve with the horizontal axis and check it against the approximate value determined in step 1 of the Procedure.

8. Experiment V: Effect of radius of torsion wire (item b). Change to the steel wire having the smaller diameter. Clamp it so that it is 0.400 m long; measure the period accurately (timing 5 or 10 vibrations); and compare it with the results from Experiment I.

Interpretation: The period is proportional to some power, say the w power, of the radius, thus

$$\frac{\text{Period}_{\text{exp. V}}}{\text{Period}_{\text{exp. I}}} = \left(\frac{\text{radius}_V}{\text{radius}_I}\right)^w \quad \text{i.e., (Period)} \propto (\text{radius})^w$$

Solve for w. This requires the use of logarithms or the log scale on your slide rule or a graph plotted on log-log paper. What do you conclude about the dependency of P on r? You will check this against the value of w obtained from a dimensional argument below in Data and Calculations.

9. Experiment VI: Effect of modulus of elasticity (items "d" and "e"). Set up the torsion pendulum using the brass wire, but otherwise with conditions the same as in Experiment I. Determine the period.

Interpretation: This datum will be used in Data and Calculations to determine whether it is the shear modulus or Young's modulus that enters into the equation for the period of the torsion pendulum.

Note: At their discretion, instructors may permit slow students to omit Experiments V and/or VI.

10. Stop here. Do not proceed farther or leave the lab until you have secured your instructor's written O.K. for your work so far.

11. Dismantle your apparatus and leave it in an orderly condition.

## DATA AND CALCULATIONS

It is now time to take stock of the situation. You have fairly definite results in your investigation of items a, b, j, k, l, and s. Probably you have acquired some insight into the things influencing the period of the torsion pendulum. If you consider carefully items n, o, p, q, and r, it is likely you will decide that they cannot influence the period (except possibly, for instance, as a change of temperature may slightly affect the properties of the wire) and hence will eliminate them from further consideration. A careful consideration of items f, g, h, and i ought to show that the important thing about the plate is its moment of inertia, but that those other items are not important except as they affect the moment of inertia. Therefore, they can be eliminated from further consideration. You might suspect also that item c, the density of the wire, is unimportant and similarly eliminate it. It remains now to investigate items d, e, m, and t.

a. Determination of rough formula by means of dimensional reasoning.

You have found out already that the period is independent of the amplitude, but that $P^2$ varies directly with the length of the wire $\ell$, and with the moment of inertia I of the plate plus the rings. Let us also suppose that $P^2$ is dependent on some power of the modulus of elasticity E of the wire--say the "x"th power. Let us also suppose it is dependent on some power of the acceleration of gravity g, say the "y"th power, as well as on the "z"th power of the radius R of the wire. An investigation of items d, e, and m will verify the result we have obtained in Experiment V. Therefore we may write:

$$P^2 = (\text{constant})\ell I E^x g^y R^z. \tag{1}$$

Both the shear modulus and Young's modulus are expressed in terms of force per unit area divided by a ratio, either the angle of shear or an elongation per unit length. The dimensions of force per unit area are $(M)(T)^{-2}(L)^{-1}$. The radius R has the dimension (L), the acceleration of gravity g the dimensions $(L)(T)^{-2}$, the moment of inertia of plate plus rings I the dimensions $(M)(L)^2$, the length of the wire $\ell$ the dimension (L), and the period P the dimension (T). Now we may substitute the proper dimensions in Equation (1) as follows:

$$(T)^2 = (L) \cdot (M)(L)^2 \cdot (M)^x(T)^{-2x}(L)^{-x} \cdot (L)^y (T)^{-2y} \cdot (L)^z \tag{2}$$

Comparing exponents for (L), we have:
    $0 = 1 + 2 - x + y + z,$
comparing exponents for (M), we have:
    $0 = 1 + x,$
comparing exponents for (T), we have:
    $2 = -2x - 2y.$

These three equations in x, y, and z may be solved by inspection, and yield the results:

    x =                    y =                    z =

If one of the quantities is zero, what does it mean? Answer: _____

Fill in the following blank with the formula for the period of the torsion pendulum as determined so far. Use the symbol E for a modulus of elasticity. We shall determine which modulus in the next part.

$$(\text{Period})^2 = (\text{constant})_____ \tag{3}$$

b. Determination of the nature of the modulus E. Using Equation (3), you may write:

$$\frac{P^2_{\text{steel}}}{P^2_{\text{brass}}} = \frac{E^x_{\text{steel}} R^z_{\text{steel}}}{E^x_{\text{brass}} R^z_{\text{brass}}} \tag{4}$$

Substituting experimental values for P and R from Experiment VI, you can obtain the ratio of the elasticities, $\frac{E_{\text{steel}}}{E_{\text{brass}}}$. The ratio of Young's modulus for steel and brass is not the same as that of the shear modulus. If your data were taken with care, you should

have a clear indication as to which modulus is involved here. Consult your text or handbook for values of the moduli of elasticity, but do not take them too literally because different samples have somewhat different moduli. Your instructor can tell you the moduli for the brass and steel wires.

The modulus involved here is _____.

c. Determination of the constant and the complete formula.

Review the discussion of the slope of a curve given in the section of the laboratory manual on Graphs. Writing Equation (3) in the form

$$(Period)^2 = [(constant) \angle E^X R^Z] I \qquad (5)$$

it is seen that the slope of the straight line of the graph of $P^2$ plotted against I, as done in Experiment IV, represents the quantity in the square brackets. Since you now know all the quantities in the square brackets except the constant, you can measure the slope on your graph and calculate the constant.

Denoting Young's modulus by Y or the shear modulus by $\eta$, and substituting the calculated value for the constant, you may write the final value of the formula for the period of the torsion pendulum as:

$$(Period)^2 = \qquad (6)$$

The wire is circular and also formulas for periods often involve $\pi$. Do you think your formula could be written more simply with $\pi$ in it? If you have another idea as to the formula, write it here:

$$(Period)^2 = \qquad (7)$$

DISCUSSION

1. Comment on why the amplitude does not affect the period (Experiment I).

2. Comment on why the weight does not affect the period (Experiment III).

3. What is the value of any quantity to the zero power? Comment on the dimensional argument in

4. As a check on your experimental discrimination between Young's modulus and the shear modulus, to determine which was involved in the torsion pendulum formula, present a good theoretical argument as to which should be involved.

5. There was not time to investigate all the items in the list of section G in Theory and Method. Suppose you are not sure if the period depends on the density of the wire and wish to find out experimentally. How would you actually do the job? Remember that different wires have different moduli of elasticity as well as different densities.

6. Although the formula, $I = \frac{1}{2}M(r_1^2 + r_0^2)$, used for the moment of inertia of the ring can be derived with calculus, and often is, it can also be derived using only algebra and geometry. Derive the formula, or justify it. Use the symbol D for the density of the ring.

7. Why was it necessary to measure the diameter of the wire in several places?

8. Automobile engineers have devoted a great deal of effort to the problem of torsional vibrations in the crankshaft of a car. What causes them? Do you suppose the engineers have tried to increase or diminish their amplitude?

9, 10, 11, 12. Discuss such things as basic ideas of the experiment, conclusions to be drawn from it, quantitative estimates of sources of errors, and possible improvements in the experiment.

Note: It was remarked in section I of the Theory and Method that the formula for the torsion pendulum can be made to sound plausible without the use of calculus. The only difficult part of the argument concerns the exponent that goes with the radius of the wire. The formula is often written

$$Period = 2\pi\sqrt{\frac{I}{t}}$$

where $t$ is the torque per radian, or torsion constant of the wire. It is not difficult to see that the torsion constant must be proportional to the fourth power of the radius of the wire from the following argument. The angle of shear $\varphi$ per radian of twist is proportional to the force per unit area since by definition
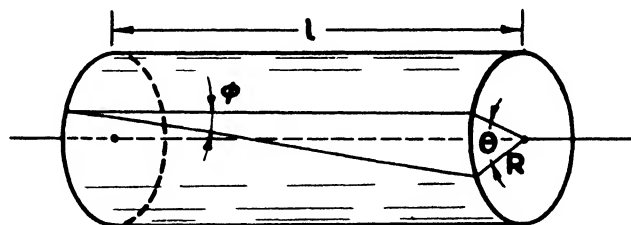


Figure 2. Horizontal Torsion Wire

of the shear modulus $\eta$, $\eta = (force/area)/\varphi$. Now $\varphi$ is inversely proportional to the radius, but the area is proportional to the radius squared, so that the force is proportional to the radius cubed; the torque is the force times the radius so that the torque is proportional to the fourth power of the radius. This argument is not rigorous, but it has the general ideas in it.

Name_____Box No._____Date in Lab_____

Partner_____Lab Section_____Instructor_____

DATA SHEET

General Measurements:

Number of torsion plate                    _____

Mass of ring with large diameter          _____

Mass of ring with small diameter          _____

Inner diameter of larger ring      _____  _____  radius _____

Outer diameter of larger ring      _____  _____  radius_____

Inner diameter of smaller ring     _____  _____  radius_____

Outer diameter of smaller ring     _____  radius_____

Moment of inertia of larger ring          _____

Moment of inertia of smaller ring         _____

Mass of plate                             _____

Diameter of plate                         _____

Approximate moment of inertia of plate    _____

Zero setting and correction to micrometer (give details) _____

Diameter of larger steel wire    _____  _____  _____  _____
                                                              Average_____

Diameter of smaller steel wire   _____  _____  _____  _____
                                                              Average_____

Diameter of brass wire           _____  _____  _____  _____
                                                              Average__ _____

Experiment I:

Length of wire _____

Which ring is on plate?_____

Which wire is used?_____

Time for _____(10 or 20?) periods with amplitude of 45$^o$_____

Time for _____periods of amplitude 90$^o$_____

Time for _____periods of amplitude 180$^o$_____

What effect does amplitude have on period?_____

Average period                                    _____

Experiment II:(Same wire and ring as for Experiment I)

|  | 20 (or 10?) Periods | Period |
|---|---|---|
| Time with length of 60 cm. | _____ | _____ |
| Time with length of 40 cm. (copy) | _____ | _____ |
| Time with length of 30 cm. | _____ | _____ |
| Time with length of 15 cm. | _____ | _____ |

What relation do you find between P and $\ell$ ?

DATA SHEET (Continued)

Experiment III:

Time for _____periods with _____kg suspended from bottom of plate by a string 0.3 m in
     length_____

Does the weight affect the period? (Yes or No)_____

Experiment IV:

Length of wire_____ Same wire as in Experiment I

|  | 10 (or 20) Periods | Period |
|---|---|---|
| Time with plate only | _____ | _____ |
| Time with smaller ring | _____ | _____ |
| Time with larger ring | _____ | _____ |
| Time with both rings | _____ | _____ |

Graphical determination of $I_0$ _____. $I_0$ calculated from step 1.

Relation of P to moment of inertia:_____

Experiment V:

Wire with small diameter. Length of wire _____. Which ring?_____

_____Periods                    _____Periods

_____           _____ Period_____

What effect does the radius seem to have on the period? _____

_____

Experiment VI:

Brass wire. Which ring on plate _____. Length of wire _____

_____Periods                    _____Periods

_____           _____ Period_____

Young's modulus for steel _____ for brass _____ ratio _____

Shear modulus for steel _____ for brass _____ ratio _____

Source of information about moduli_____

Ratio determined experimentally _____ _____

Which modulus appears in the formula?___ _____

Other data or remarks:

Approval (Procedure step 10)_____

Hours spent on this experiment: Study_____Lab_____Report_____

EXPERIMENT 51-10

WAVE MOTION AND SOUND

## OBJECT

I. To study wave motion in a string.

II. To study characteristics of vibrating strings.

III. To study conditions for resonance of an air column.

## APPARATUS

In Lab: I. Mechanical rotator with oscillator attachment, assorted 10-foot lengths of string including samples, analytical balance, analytical weights, slotted weights, weight holders, pulleys on tripod table supports, clock, 2-meter stick. II. Special sonometer, 2 steel wires for sonometer plus samples, micrometer, 3 tuning forks, meter stick. III. Calibrated audio oscillator, earphone in mount, wires, assorted lengths of telescoping cardboard tubing, meter stick, tuning forks, one thermometer in lab.

## THEORY AND METHOD

I. A. It is shown in lecture that the velocity $v$ of any kind of wave is equal to the frequency $f$ multiplied by the wavelength $\lambda$ ; that is

$$v = f\lambda \qquad * \quad (1)$$

For the case of waves traveling in a cord having a mass $m_1$ per unit length, and under a tension $T$, it is shown that

$$v = \sqrt{\frac{T}{m_1}} \qquad * \quad (2)$$

If the cord is to vibrate in resonance with the driving frequency $f$, then there must be an integral number $n$ of half wavelengths in the length $\ell$ of the cord; that is

$$\ell = \frac{n\lambda}{2} . \qquad * \quad (3)$$

Combining (1), (2), and (3) one concludes that

$$f = \frac{n}{2\ell}\sqrt{\frac{T}{m_1}} \qquad * \quad (4)$$

B. The mechanical oscillator does not produce plane waves in the cord, but rather it forms circular waves (so-called circularly polarized waves). Equations (1), (2), (3), and (4) are valid for either plane waves or circular waves, since the latter are merely two plane waves compounded at right angles, $90°$ out of phase with each other.

C. One or more strings are fastened to the mechanical oscillator and run over pulleys as shown in Figure 1. Various weights may be placed on weight hangers tied to the end of the strings. One may vary the driving frequency, the length of the string, or the tension in it; and, by using different strings, one may observe the effect on the results of the mass per unit length of the string.

II. D. The experiment on the sonometer shown in Figure 2 is fundamentally similar to the experiment with the waves in the cord except that the frequency is high enough to be audible and may be compared by ear with a tuning fork. In the fundamental mode of vibration there is only a single loop in the string so that its length is $\frac{\lambda}{2}$ and $n = 1$. Hence Equation (4) becomes:

$$f = \frac{1}{2\ell}\sqrt{\frac{T}{m_1}}$$

One may vary the length $\ell$ and tension $T$ of a given wire, and by using different wires may check the influence on the frequency $f$ of the mass per unit length $m_1$.
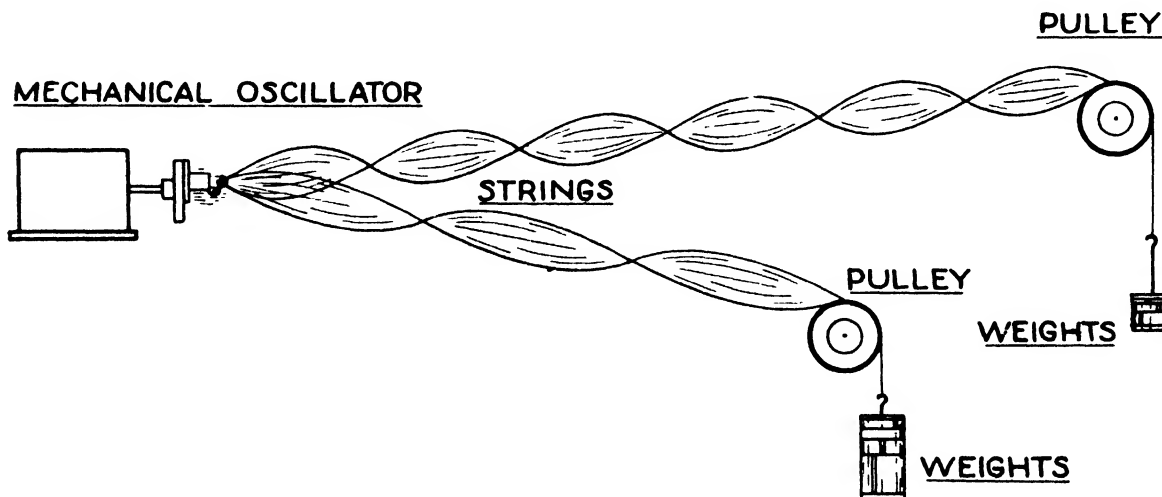


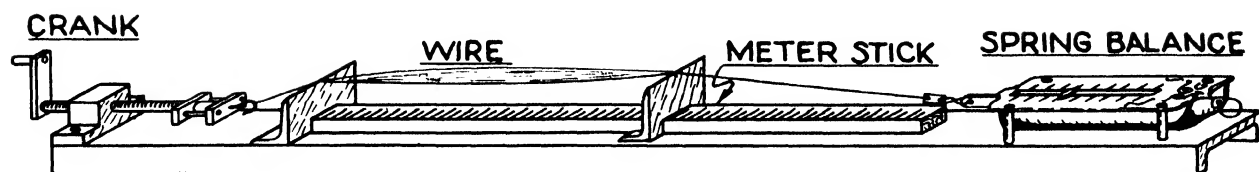Figure 1. Waves in strings produced by a mechanical oscillator.

Figure 2. The Sonometer

III. E. An air column of proper length resonates to a note of given frequency and apparently amplifies it. (Actually more power is drawn from the source when the column is in resonance.) If the column is open at an end, there must be a (displacement) loop at the end, while if the end is closed, there must be a (displacement) node at the end. The air column of the present experiment is confined by a cardboard tube. The column is open at one end, and is closed at the other end. A telephone receiver at the closed end sets the air column to vibrating when the earphone is energized by a calibrated electrical audio oscillator, see Figure 3. The frequency may be read from its dial, or the frequency may be determined by comparison with the frequency of a tuning fork.

F. One might suppose there would be a displacement loop at the earphone end of the tube because the motion of the earphone diaphragm sets the air column vibrating. Actually, whether there is a node or loop the driving end depends on what is used at the closed end to set the air column vibrating. The earphone used in this experiment is so "solid" compared with the "soft" air in the air column that the earphone acts as a closed end. Therefore, there is a displacement node at the earphone. (To be precise, the displacement node is located within a fraction of a millimeter from the earphone, and for practical purposes may be said to be at the earphone.)

G. At the open end of the air column there is a displacement loop which is a short distance beyond the end of the tube. (This distance is 0.6133R where R is the radius of the tube--see Levine and Schwinger, Bulletin of the American Physical Society, B10, Stanford Meetings, July 1947. The reasons why the loop is not exactly at the end are too involved for a discussion here. Loosely speaking, you may say the sound has to get a little way outside the tube before it really knows it is outside.)

H. With the exceptions noted in the preceding two paragraphs, the closed tube will resonate when its length is $\frac{\lambda}{4}$, $\frac{3\lambda}{4}$, $\frac{5\lambda}{4}$, $\frac{7\lambda}{4}$, etc. The difference in length of tube that resonates a particular note is, however, accurately $\frac{\lambda}{2}$.

I. In courses on heat it is shown that the velocity V (in $\frac{m}{sec}$) of sound in a gas is given by

$$V = \frac{\gamma p}{d} \qquad (6)$$

where p is the pressure (in $\frac{newtons}{m^2}$), d is the density (in $\frac{kg}{m^2}$), and $\gamma$ (Greek letter Gamma) is a pure number (the ratio of the specific heat of the gas at constant pressure to the specific heat at constant volume), that is 1.41 for air. For ordinary temperatures Equation (6) can be juggled to the form

$$V = 331.3 \left(1 + \frac{t}{546}\right) \frac{meters}{sec} \qquad (7)$$

where t is the temperature in degrees Centigrade.

PROCEDURE

The instructor should divide the class into groups, some starting with I, others with II, and others with III. Since this is the last experiment of the quarter, it may be necessary to hand in the experiment at the end of the lab period or shortly afterward. For this reason the instructor may direct you to omit or to shorten considerably some parts.

I. 1. Connect one of the heavier strings to the mechanical oscillator, run the string over a pulley and tie a weight holder to it. Place about 125 grams on the end of the string (including the weight of the hanger). Measure the length of the string from oscillator to pulley. Vary the frequency of rotation of the mechanical oscillator until you get the string to vibrate in a resonant condition; that is, with a definite number of clearly defined loops of considerable amplitude. Determine the frequency by using the revolution counter to count the number of revolutions in, say, 20 seconds.
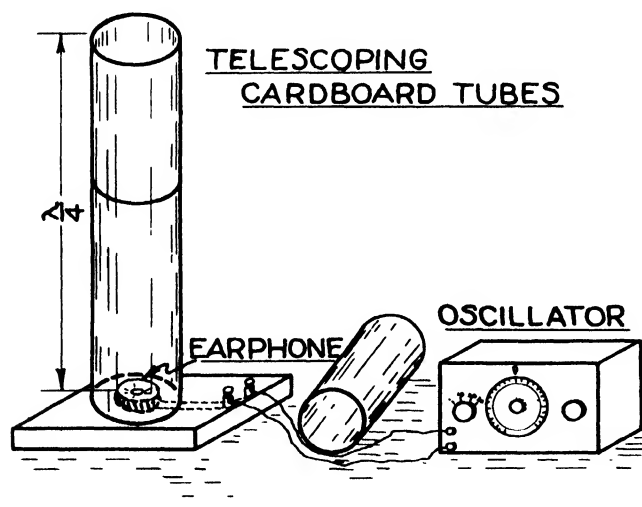


Figure 3. The air column resonance experiment

Note: It is not always possible to get the resonances as sharply defined as one would like, but some care in adjusting the frequency usually helps matters.

2. Vary the frequency to observe as many other clearly defined resonances as possible. (You will be able to observe resonances for one loop, two loops, three loops, and so on, perhaps up to as many as ten loops--depending on the length of the string, its mass per unit length, and the tension.)

3. Repeat steps 1 and 2, but with the tension changed to values of 50 grams weight and then to 300 grams weight.

4. If you are not convinced that for a particular value of tension, frequency, and mass per unit length, the number of loops is proportional to the length of the string, try an experiment in which you vary the length of the string.

5. Determine the mass of the string used (or one like it) on an analytical balance, and by measuring its length, calculate the mass per unit length.

6. Repeat steps 1, 2, and 5 with the lighter string. It is not necessary to remove the heavier string.

II. 7. Set up the sonometer using the heavier of the two wires supplied. Set the length of the string at about 25 cm, and gradually increase the tension (by turning the crank) as you pluck the string until the pitch is the same as that of the lowest-frequency tuning fork you have. Record the tension. Increase the tension gradually until the pitch is the same as that of the next-lowest-frequency fork you have. Continue in this way until you have varied the tension from minimum to maximum value.

8. With the tension corresponding to resonance with the fork of highest pitch, now investigate the effect of increasing the length of the wire. Record the lengths necessary to give resonance with the other forks you have.

9. Finally change to the other wire and adjust the length and tension so that you get some clearly defined resonance.

10. Determine the mass of the wires used (or ones like them) on an analytical balance, and, by measuring their length, determine their mass per unit length. If you prefer, you may measure the diameter of the wires, their lengths, and then calculate the mass per unit length, using the known density of steel.

11. Qualitatively (or quantitatively if you prefer) investigate the frequency ratios of various musical intervals by changing the length of the wire. For instance, set the tension so the wire has the same frequency as some fork. Then vary the length of the wire so as to get intervals such as a major tone, a minor third, a major third, a fourth, a fifth, etc. (If you don't "have an ear for music" omit this part.) To prevent warping of the sonometer base, loosen the wires when you are finished.

III. 12. Set the oscillator for a frequency of about 500 $\frac{cy}{sec}$. (Compare it with a tuning fork if necessary.) With the telescoping cardboard tubes, starting with a short length, determine the lengths of the tube that

will give clearly defined sharp resonances. (Do not increase the tube length to more than about 6 feet.)

13. Repeat for at least two other frequencies from about 250 to 1000 $\frac{cy}{sec}$. (If the forks do not have high enough frequencies, you can still do the experiment by setting an uncalibrated oscillator properly by tuning to an octave, for instance.)

14. Measure the diameter of the tubes.

15. Measure the room temperature.

DATA AND CALCULATIONS

I. a. From Equation (4) calculate the frequencies corresponding to your measurements in steps 1 - 6 of the Procedure. Enter these calculated frequencies in the appropriate columns on the Data Sheet where they may be compared with the observed frequencies. Be sure to use proper units--Equation (4) contains both force units and mass units.

II. b. From Equation (5) calculate the frequencies corresponding to your measurements in steps 7 - 11 of the Procedure. Enter these calculated frequencies in the appropriate columns on the Data Sheet where they may be compared with the observed frequencies. Be sure to use proper units.

III. c. From Equations (1) and (7), and information in section H of the Theory and Method, determine the lengths of closed air column that would be expected to resonate to the frequencies you used in steps 12 - 15 of the Procedure. Enter these lengths in appropriate blanks on the Data Sheet where they may be compared with the observed lengths.

d. The observed and calculated lengths will disagree because of the end effects discussed in sections F and G. You should anticipate a constant difference between calculated and observed lengths. This difference may be compared with that to be expected from the discussion in section G.

REPORT

1. In this experiment you have observed a large number of conditions for resonance between a driving source (the mechanical or electrical oscillator) and a driven mechanism (the string or air column). You should note also that the driven mechanisms absolutely do not resonate to any frequency, but only to certain ones. Some things (certain kinds of electrical circuits) resonate to only a single frequency. Can you name a common household instrument that may be tuned to resonate at a desired frequency? What drives it? Why is it important that it not resonate to several different frequencies for a given setting?

2. How well do your calculated and observed frequencies agree?

3. How well do your calculated and observed lengths agree? What seems to be the magnitude of the end correction?

4, 5, 6, 7. Discuss the basic ideas illustrated by the experiments, conclusions to be drawn from them, quantitative estimates of sources of error, and possible improvements in the experiments.

EXPERIMENT 51-10, WAVE MOTION AND SOUND                                    4
DATA SHEET

1. Weight on end of string (including hanger)_____

Length of string from oscillator to pulley        _____

No. of loops_____Time for _____rev._____Frequency _____Calculated Freq._____

| | | | | |
|---|---|---|---|---|
| _____ | _____ | _____ | _____ | _____ |
| _____ | _____ | _____ | _____ | _____ |
| _____ | _____ | _____ | _____ | _____ |
| _____ | _____ | _____ | _____ | _____ |

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

3. Weight on end of string including hanger _____

Length of string from oscillator to pulley        _____

No. of loops_____Time for _____rev._____Frequency _____Calculated Freq._____

| | | | | |
|---|---|---|---|---|
| _____ | _____ | _____ | _____ | _____ |
| _____ | _____ | _____ | _____ | _____ |
| _____ | _____ | _____ | _____ | _____ |
| _____ | _____ | _____ | _____ | _____ |

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

Weight on end of string including hanger _____

Length of string from oscillator to pulley _____

No. of loops_____Time for _____rev._____Frequency _____Calculated Freq._____

| | | | | |
|---|---|---|---|---|
| _____ | _____ | _____ | _____ | _____ |
| _____ | _____ | _____ | _____ | _____ |
| _____ | _____ | _____ | _____ | _____ |
| _____ | _____ | _____ | _____ | _____ |

Determination of mass per unit length of string used in experiments recorded on this page.

Mass_____Length_____Mass per unit length_____

Other data:

## DATA SHEET (Continued)

6. Weight on end of string including hanger_____

Length of string from oscillator to pulley  _____

No. of loops_____Time for _____rev._____Frequency _____Calculated Freq._____

_____     _____  _____     _____       _____

_____     _____  _____     _____       _____

_____     _____  _____     _____       _____

_____     _____  _____     _____       _____

Determination of mass per unit length of string used in experiments recorded on this page.

Mass_____Length_____Mass per unit length_____

Other data:

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

7. Length of string_____

Tension_____Frequency_____Calculated Frequency_____

_____     _____     _____

_____     _____     _____

_____     _____     _____

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

8. Tension_____

Length _____Frequency_____Calculated Frequency_____

_____     _____     _____

_____     _____     _____

_____     _____     _____

Mass of wire used in parts 7 and 8_____Length_____

Zero correction of micrometer_____

Diameter of wire used_____  _____  _____  _____ Density_____

Mass per unit length_____

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

9. Tension_____Frequency _____Calculated Frequency_____

   Length_____

   Mass of wire used in part 9 _____ Length_____

   Diameter of wire used_____  _____  _____  _____ Density_____

   Mass per unit length_____

11. Other data:

12. Observed frequency_____

     Calculated lengths  _____   _____   _____   _____   _____

     Observed lengths   _____   _____   _____   _____   _____

     Difference in length_____   _____   _____   _____   _____

     Observed frequency_____

     Calculated lengths  _____   _____   _____   _____   _____

     Observed lengths   _____   _____   _____   _____   _____

     Difference in length_____   _____   _____   _____   _____

     Observed frequency_____

     Calculated lengths  _____   _____   _____   _____   _____

     Observed lengths   _____   _____   _____   _____   _____

     Difference in length_____   _____   _____   _____   _____

     Observed frequency_____

     Calculated lengths  _____   _____   _____   _____   _____

     Observed lengths   _____   _____   _____   _____   _____

     Difference in length_____   _____   _____   _____   _____

14. Diameter of cardboard tube_____

15. Room temperature_____

     Velocity of sound in $\frac{m}{sec}$ at this temperature_____

Approval_____

Hours spent on this experiment: Study_____Lab____   _____Report_____